GYROSTAT ATTITUDE CONTROL USING NONLINEAR SDDRE METHOD

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ABSTRACT

This paper deals with the design of nonlinear attitude control law for satellite based on the State Dependent Differential Riccati Equation (SDDRE) technique. A satellite model with momentum exchange devices is used which is named gyrostat. For singularity avoidance, quaternion representation of satellite orientation is used. The proposed control law is capable to generate 3-D attitude maneuvers for tracking a reference attitude profile.

1. INTRODUCTION

New earth observation missions place stringent requirements on the satellite attitude control system (ACS) in terms of fine pointing and rapid target acquisition. This means that ACS should be capable to generate 3-D maneuvers as well as high speed slew rates. The design of such ACS is a complex task as a result of coupled nonlinear dynamics. This paper examines the use of SDDRE method as a nonlinear tracking attitude control law for a satellite containing reaction wheel control devices.

2. GYROSTAT EQUATION OF MOTION

Consider a rigid spacecraft with an N-Wheel cluster installed to provide internal torques. Then the rotational equations of motion for the satellite can be expressed as [1]:

\[ [J]\dot{\theta} = -\dot{\theta} \times [J] \theta - \theta \times [G] h_a - [G] u_a + u_e \]

(1)

\[ h_a = u_a \]

(2)

Where \( h_a \) is the \( N \times 1 \) vector of the axial angular momenta of the reaction wheels, \( \theta \) is the angular-velocity vector of the satellite, \( u_s \) is the \( 3 \times 1 \) vector of external torques, \( u_a \) is the \( N \times 1 \) vector of the internal torques applied by the platform to the wheels and \( [G] \) is the \( 3 \times N \) matrix whose columns contain the axial unit vectors of the N wheels. \( [J] \) is an inertialike matrix defined as \( [J] = [I] - [G][I_s][G]^{T} \). Where \( [I] \) is the moment of inertia of the satellite, including the wheels, and \( [I_s] = \text{diag}(I_{s1}, I_{s2}, \ldots, I_{sN}) \) is a diagonal matrix with the axial moment of inertia of the wheels. Superscript \( \times \) represents the skew symmetric matrix operator. To derive mathematical model, minimal representation of
attitude kinematics, such as Euler angels or Rodrigues parameters, contains singularities and are hence not well suited for the design of global control algorithms. For this reason, global representation of satellite kinematics, such as quaternions, is used to ensure that the control objectives are met over a large range of operating conditions [2]. The kinematics of motion can be described by quaternion vector \( \mathbf{q} \) where

\[
\begin{bmatrix}
-\beta_1 \\
-\beta_2 \\
-\beta_3 \\
-\beta_4
\end{bmatrix}
\]

\( \mathbf{B} + 1 \) and

\[
\dot{\mathbf{q}} = \frac{1}{2} [\mathbf{B}] \mathbf{q}
\]

\( [\mathbf{B}] = \begin{bmatrix}
-\beta_1 & -\beta_2 & -\beta_3 & -\beta_4 \\
\beta_0 & -\beta_3 & \beta_2 & 0 \\
\beta_3 & \beta_0 & -\beta_1 & 0 \\
-\beta_2 & \beta_1 & \beta_0 & \beta_1 \\
\end{bmatrix}
\] (3)

3. STATE-DEPENDENT-DIFFERENTIAL-RICCATI-EQUATION-CONTROL

State Dependent Riccati Equation (SDRE) method [3] is a recently emerging nonlinear control system design methodology for direct synthesis of nonlinear feedback controllers. By turning the equations of motion into a linear-like structure, this approach permits the designer to employ linear optimal control methods such as the LQR and H\(_\infty\) design technique for the synthesis of nonlinear control systems. The method is based on “extended linearization” of the process dynamics.

Suppose that the dynamic model of system is expressed by:

\[
\dot{x} = f(x) + B(x)u, \quad x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^r
\]

(4)

To track a reference trajectory \( y_{ref} \) the performance index is defined as:

\[
J = \frac{1}{2} \int_0^\infty (\mathbf{y} - \mathbf{y}_{ref})^T Q(x)(\mathbf{y} - \mathbf{y}_{ref}) + u^T R(x)u \, dt
\]

(5)

The SDRE approach assumes that the dynamic model (4) can be placed in the state-dependent-coefficient form \( \dot{x} = A(x)x + B(x)u \). Where \( f(x) = A(x)x \) (the choice of the matrix \( A(x) \) is not unique). Then it is proceeded as in the LQT case, and a state-feedback control is obtained in the form of \( u(x) = -R^{-1}(x)B^T(x)(-P(x)x - S(x)), \) where the \( P(x) \) is the unique, symmetric positive definite solution of the state dependent algebraic Riccati equation:

\[
A^T(x)P(x) + P(x)A(x) + Q(x) - P(x)B(x)R(x)^{-1}B^T(x)P(x) = 0
\]

(6)

The \( S(x) \) is calculated from:

\[
(A(x) - B(x)R(x)^{-1}B^T(x)P(x))^T S(x) + C(x)^T Q(x)y_{ref} = 0
\]

(7)

The requirements for this method are [4]: i) The full state estimation is available. ii) \( f(x) \) is continuously differentiable and \( f(0)=0 \). iii) \( B(x) \) is smooth and \( B(x) \neq 0 \) \( \forall x \) iv) \( Q(x), R(x) \) are positive definite \( \forall x \) v) The pair \( (A(x),B(x)) \) is pointwise stabilizable \( \forall x \) \ where \( \text{rank}[M_{cf}(x)] = \text{rank}[B(x) A(x)B(x) A(x)^2 B(x)\cdots A(x)^{n-1} B(x)] = n \) \( \forall x \).

There are two drawbacks with the SDRE method. First, online solving the ARE (6) is computationally intensive, especially for higher order systems such as gyrostat. Second, SDRE method imposes an overly restrictive requirement on the pointwise controllability.
of the system. There are many nonlinear systems which this condition does not hold but the system is physically controllable (such as gyrostat attitude control system).

Haessig et al. [5] offer an alternative to the SDRE method which addresses the issues of high computational requirements and the potentially overly restrictive controllability requirements. The key idea underlying this new method is the use of differential (rather than algebraic) Riccati equation (SDDRE), and their real time solution by numerical integration. Then the resulting differential equations for calculating $P(x)$ and $S(x)$ are:

\[
\dot{P}(x) = A^T(x)P(x) + P(x)A(x) + Q(x) - P(x)B(x)R(x)^{-1}B(x)^TP(x)
\]

\[
\dot{S}(x) = (A(x) - B(x)R(x)^{-1}B(x)^TP(x))^T S(x) + C(x)^T Q(x) y_{ref}
\]

The use of SDDRE does not require local controllability, and this is examined that the computational load of the SDDRE is little compared to SDRE [5]. There are some characteristics that make the use of SDDRE control favorable, such as simplicity, sub-optimality, robustness and flexibility (especially in choosing the state dependent weight matrices $Q(x)$ and $R(x)$) [6].

4. SDDRE ATTITUDE CONTROL OF GYROSTAT

Using the dynamic model described in section 2, we form the following state dependent factorization:

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{h}_a
\end{bmatrix} = 
\begin{bmatrix}
-\hat{J}\hat{J}^{-1}\beta \\
0.5\hat{B}(\beta)\hat{J}^{-1}
\end{bmatrix}
\begin{bmatrix}
\beta \\
h_a
\end{bmatrix} + 
\begin{bmatrix}
0_{3x4} \\
0_{3x4}
\end{bmatrix}
\begin{bmatrix}
0_{4xN} \\
0_{4xN}
\end{bmatrix}
\begin{bmatrix}
\dot{u}_a \\
\dot{y}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{h}_a
\end{bmatrix} = 
\begin{bmatrix}
\hat{J}\hat{J}^{-1}\beta \\
\hat{B}(\beta)\hat{J}^{-1}
\end{bmatrix}
\begin{bmatrix}
\beta \\
h_a
\end{bmatrix} + 
\begin{bmatrix}
0_{3x4} \\
0_{3x4}
\end{bmatrix}
\begin{bmatrix}
0_{4xN} \\
0_{4xN}
\end{bmatrix}
\begin{bmatrix}
\dot{u}_a \\
\dot{y}
\end{bmatrix}
\]

In simulation we consider a satellite with a four-wheel cluster. The inertia matrix for the satellite and the wheels characteristics are as follows:

\[
I = \begin{bmatrix}
200 & 0 & 0 \\
0 & 200 & 0 \\
0 & 0 & 175
\end{bmatrix} \text{kg.m}^2, \quad G = \begin{bmatrix}
1 & 0 & \sqrt{3}/3 \\
0 & 1 & \sqrt{3}/3 \\
0 & 0 & 1
\end{bmatrix}, \quad I_s = 0.338 \text{ kg.m}^2
\]

Further, the following initial conditions are chosen for simulations.

\[
(\theta) = \begin{bmatrix}
2.86, -1.15, 2.29
\end{bmatrix} \text{deg/sec}, \quad \beta(0) = [0.99, -0.12, 0.04, 0.10]^T
\]

\[
\Omega_{\text{wheel}}(0) = [2309.4, 2309.4, 2309.4, -4000]^T \text{ rad/sec}
\]

The reference trajectory is chosen as: $\beta_{\text{ref}} = [0.29, 0.64, 0.29, -0.64]^T$ which corresponds to a 3-2-1 Euler axis rotation of $\theta_1 = -75\text{deg}, \theta_2 = 90\text{deg}, \theta_3 = 55\text{deg}$. We select the penalty matrices as $Q = \text{diag}(20, 20, 20, 20)$, $R = \text{diag}(1, 1, 1, 1)$. We select the penalty matrices as $Q = \text{diag}(20, 20, 20, 20)$, $R = \text{diag}(1, 1, 1, 1)$.

Simulation results are shown in Fig. 1. The results show that by using an acceptable control effort, the performance of the reorientation maneuver is completely satisfactory. The CPU time used to compute the SDDRE control law in each time step is shown in Fig. 1 (the simulating computer CPU is a Pentium III, 800 MHz). It indicates that the controller CPU time is much less than the controller sampling time (which is 0.3 sec).

Overall, the results show that the SDDRE controller is very effective in reorienting the gyrostat.
5. REFERENCES


