Model and controller reduction

Model reduction

Model order reduction is of fundamental importance in many modelling and control applications. The most frequently employed model reduction algorithms for linear, **stable**, continuous- or discrete-time systems are the balancing related *absolute error* methods like the **balanced truncation approximation** (BTA), **singular perturbation approximation** (SPA), and **Hankel-norm approximation** (HNA). Numerically reliable accuracy enhanced versions of the BTA and SPA methods have been proposed in [1] and [2]. These so-called **balancing-free square-root** (BFSR) methods rely on computing well-conditioned truncation matrices using exclusively square-root information, i.e. the Cholesky factors of gramians. The basic methods in combination with coprime factorization [3] or spectral decomposition techniques [4], can be used to reduce **unstable** systems as well.

Alternatively, a *relative error* method like the **balanced stochastic truncation** (BST) method can be used to obtain uniform approximations of the frequency-response over the whole frequency range, preserving also the phase information. For example, by employing the BST approach, approximations of minimum-phase systems result minimum-phase, which is generally not true when employing absolute error methods. A BFSR-version of the BST method has been proposed in [5] and the BST-method has been extended in [6] to handle the case of systems with zeros at infinity, as for example, strictly proper systems. The enhanced BST method relies on computing a left spectral factor using a general inner-outer factorization method.

Controller reduction

To address *controller reduction* problems, **frequency-weighted** extensions of the three basic methods BTA, SPA and HNA can be employed. Theoretical and computational enhancements of the frequency-weighted BTA and SPA methods have been proposed in [7,16]. These methods have been extended to solve efficiently a class of stability/performance preserving controller reduction problems [12,16], the stability preserving frequency-weighted coprime factor controller reduction problem [14], as well as the performance preserving frequency-weighted coprime factor H-infinity controller reduction problem [15]. Several limitations of the frequency-weighted HNA have been removed by developing new projection formulas based on efficient descriptor system computational techniques [8]. Optimal feed-through matrices minimizing L-infinity weighted approximation error norms can be computed using fast methods based on outer approximations, thus circumventing the need to solve large order *linear matrix inequalities* [9].

Numerical software

High quality numerical software for model and controller reduction is available in RASP-MODRED, a collection of LAPACK-based Fortran routines from the RASP library for solving computational problems appearing in the context of model and controller reduction. It includes implementations of the whole suite of the new generation of accuracy enhancing **balancing-free square-root** methods for the BTA, SPA, HNA, BST, as well as for several frequency-weighted extensions of these methods [10,12]. The implemented algorithms are generally superior to those implemented in the model and controller reduction tools of commercial packages (e.g. MATLAB or MATRIX-X). The model reduction tools have been successfully employed to reduce dense systems up to an order of 5000 on desktop computers (see NICONET Newsletter 9, pp. 13-16). The RASP-MODRED routines cover the reduction of both stable and unstable, as well as continuous- and discrete-time systems and controllers. The whole collection of RASP-MODRED routines is freely available via the SLICOT library. The collection of model reduction routines available in SLICOT is complemented by several gateway functions allowing a convenient user-friendly operation via MATLAB or Scilab. Detailed descriptions of the available model and controller reductions tools can be found in [11] and [13], respectively.

Related publications:

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