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A New Algorithm for Three-finger Force-closure Grasp of Polygonal Objects

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Abstract—**We prove a new necessary and sufficient condition for 2D three-finger equilibrium grasps and implement a geometrical algorithm for computing force-closure grasps of polygonal objects in this article. The algorithm is quite simple and only needs some algebraic calculations. An easily computable measure of how far a grasp is from losing force-closure is provided as well. Finally, we implement the algorithm and demonstrate its usefulness by an example.**

I. INTRODUCTION

Research has been directed towards the design and control of multifingered dextrous robot hand to increase robot dexterity and adaptability.

A main character of multi-finger stable grasp is force-closure, such that the contact forces exerted by the fingers can balance arbitrary force and torque exerted on the grasped object [1, 2, 3]. Salisbury and Roth [4] have demonstrated that a necessary and sufficient condition for force-closure is that the primitive contact wrenches resulted by contact forces positively span the entire wrench space. Nguyen [5] proposed a simple test algorithm for two-finger force-closure grasps. He characterized two-finger force-closure grasps by the fact that the line joining the contact points must lie within the friction cones at these points. Ponce and Faverjon[6] developed several sufficient conditions for three-finger equilibrium grasps of polygonal objects. The equilibrium of three-finger grasp was achieved if the suface normals at three contact points positively span the plane and the intersection of the three friction cones at these points was not empty or the intersection of the three double-sided friction cones is not empty with angles between any two normals less than $\pi - 2\alpha$ (α is half-angle of the friction cones). An algorithm has been implemented based on linear programming and variable elimination in their paper. Since the conditions are not necessary, there are always force-closure grasps that don't satisfy the conditions.

The quantitative test for force-closure grasps provides a measure of how far a grasp is from losing force-closure. Trinkle [7] formalized the quantitative test as a linear programming (LP) problem by checking the existence of any positive null vector of the primitive contact wrench matrix. Chen and Burdick [8] gave an algorithm for 2D force-closure grasps based on the convex hull of the wrenches generated by the point contact friction cone edge vectors. Ferrari and Canny [9] developed a quantitative test for force-closure using the radius of a maximal ball centered at the origin and included in the convex hull of the primitive wrenches as a measure of goodness of the grasp. Liu [10] formalized qualitative test of 3-D force- closure grasps as a LP problem based on the duality between convex hulls and convex polytopes.

In this paper, a new necessary and sufficient condition for three-finger equilibrium grasps has been proven, and an easily computable algorithm for force-closure grasps of 2D objects has been implemented which efficiently reduces the amount of computation required. With the algorithm, a measure of how far a grasp is from losing force-closure can be easily computed.

II. ALGORITHM FOR EQUILIBRIUM AND FORCE-CLOSURE GRASPS

A. Relative Notions

We restrict our attention to 2D case and assume Coulomb friction now. Under Coulomb friction, a contact force is constrained to lie in a friction cone centered about the internal surface normal at contact point with half-angle α. The tangent of the angle α is called the friction coefficient. As shown in Fig.1, a friction cone at C_1 is bounding by vectors n_{11} and n_{12}, and any force f_1 is a nonnegative combination of these two vectors.

Wrench: A force f and a moment m can be combined into a wrench $w = (f, m)^T \in \Re^k$, with $k = 3$ in the case of plannar mechanics, and $k = 6$ in the case of spatial mechanics.

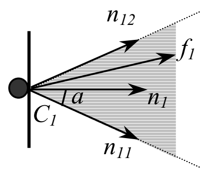


Fig.1. Coulomb friction.

Force-closure: A grasp achieves force-closure when it can resist arbitrary forces and torques.

There are other definitions of force (or form) closure, but this one is more useful for our deduction.

Equilibrium: A set of n wrenches $w_1, \cdots, w_n$ is said to achieve equilibrium when the convex hull of the points $w_1, \cdots, w_n$ in $\Re^k$ contains the origin.

Mishra, Schwartz and Sharir [3] have shown that a necessary and sufficient condition for a system of wrenches to achieve force closure is that the origin of $\Re^k$ lies in the interior of the convex hull of the primitive wrenches. In particular, force closure implies equilibrium but there are wrench systems that achieve equilibrium but not force-closure. Proposition 1 makes clear the relationship between three-finger equilibrium and force-closure grasps.

***Proposition** 1*: A two-dimensional, three-finger grasp that achieves nonmarginal equilibrium also achieves force-closure.

The proof of Proposition 1 can be found in [6]. We'll extend an algorithm for computing force-closure grasps with Proposition 1 in section 3.

Unit force vector: A unit force vector is a unit vector that is in the same direction as a contact force.

For example, contact force f_1 can be expressed as $f_1 = an_{f1}$, where $a > 0$, and n_{f1} is the unit force vector of f_1.

B. Necessary and Sufficient Condition for Equilibrium

Consider three hard fingers grasp a 2-D object and assume point contact with friction. The contact points are C_1, C_2, and C_3 (refer to Fig.2). The normal at the contact points (point to the inside of the object) are n_1, n_2, and n_3 respectively. The unit vectors n_{11}, n_{12}, n_{21}, n_{22}, n_{31} and n_{32} bound the three friction cones in pairs. External force F and moment M acts on the object on a point *O*.

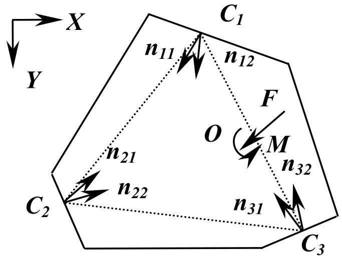


Fig. 2 Frictional three-finger 2D grasps.

Proposition 2: A necessary and sufficient condition for the existence of three nonzero contact forces, not all of them being parallel, which achieve equilibrium is that there exist three forces in the friction cones at the contact points which positively span the plane and whose lines of action intersect at some point.

See [6] for a proof of this proposition. The proposition seems to be well accepted, and there are various forms of this result in the literature [2], [3].

However there is hardly an algorithm for equilibrium grasps using Proposition 2 directly, due to the large amount of computation in searching for three contact forces satisfying the condition of Proposition 2.

We'll substitute the boundary vectors n_{11}, n_{12}, n_{21}, n_{22}, n_{31}, n_{32} of the three friction cones for the unknown contact forces in Proposition 2, and prove a new proposition that suitable to be used in computing three-finger equilibrium and force-closure grasps.

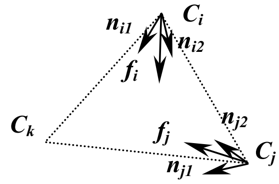


Fig. 3 Moment equilibrium of contact forces f_i and f_j.

First of all, we need a disposition to the three friction cones. Consider the three-finger equilibrium grasp shown in Fig. 3, in order to achieve moment equilibrium the contact forces f_i and f_j must satisfying that,

$$\overrightarrow{C_k C_i} \times f_i + \overrightarrow{C_k C_j} \times f_j = 0 \qquad (1)$$

The friction cone at contact point C_j is divided into two parts by line $\overrightarrow{C_j C_k}$. Since the direction of moment $\overrightarrow{C_k C_i} \times f_i$ is clockwise, the contact force f_j must lie in the region between n_{j2} and $\overrightarrow{C_j C_k}$. Otherwise there won't be contact forces f_i and f_j satisfying (1). Since the contact force f_j cannot lie in the region between n_{j1} and $\overrightarrow{C_j C_k}$, that is to say this region has no contribution to the equilibrium, we can substitute $\frac{\overrightarrow{C_j C_k}}{\left|C_j C_k\right|}$ for n_{j1} and don't change the result of equilibrium computation.

The direction of a nonzero 2D moment is either clockwise

$$\begin{cases} n_{j1} = \begin{cases} \overrightarrow{C_j C_k} / \left|C_j C_k\right| & (\overrightarrow{C_j C_k} \text{ lies in the positive cone}) \\ \overrightarrow{C_k C_j} / \left|C_j C_k\right| & (\overrightarrow{C_j C_k} \text{ lies in the negetive cone}) \end{cases} & \text{If } Sgn(\overrightarrow{C_k C_j} \times n_{j1}) \cdot Sgn(\overrightarrow{C_k C_i} \times n_{i1}) > 0 \\ n_{j2} = \begin{cases} \overrightarrow{C_j C_k} / \left|C_j C_k\right| & (\overrightarrow{C_j C_k} \text{ lies in the positive cone}) \\ \overrightarrow{C_k C_j} / \left|C_j C_k\right| & (\overrightarrow{C_j C_k} \text{ lies in the negetive cone}) \end{cases} & \text{If } Sgn(\overrightarrow{C_k C_j} \times n_{j2}) \cdot Sgn(\overrightarrow{C_k C_i} \times n_{i1}) > 0 \\ n_{j1} = n_{j2} = 0 \quad \text{If } Sgn(\overrightarrow{C_k C_j} \times n_{j1}) \cdot Sgn(\overrightarrow{C_k C_i} \times n_{i1}) > 0 \text{ and } Sgn(\overrightarrow{C_k C_j} \times n_{j2}) \cdot Sgn(\overrightarrow{C_k C_i} \times n_{i1}) > 0 & \end{cases} \qquad (3)$$

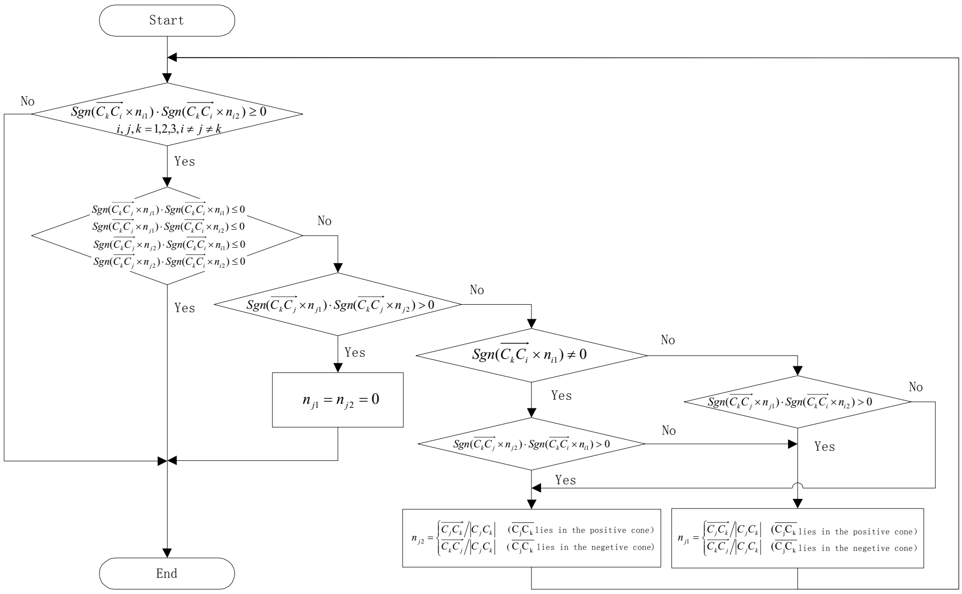


Fig. 4 Flowchart of the disposition H to the friction cones.

or anti-clockwise. Denotes the direction of a moment M by symbol $Sgn(M)$, we have

$$Sgn(M) = \begin{cases} 1 & M > 0 \\ -1 & M < 0 \\ 0 & M = 0 \end{cases} \qquad (2)$$

Define the disposition H to the friction cones: when there are $Sgn(\overrightarrow{C_kC_i} \times n_{i1}) \cdot Sgn(\overrightarrow{C_kC_i} \times n_{i2}) \ge 0$, where $i, j, k = 1,2,3$, $i \ne j \ne k$, and without losing generality suppose $Sgn(\overrightarrow{C_kC_i} \times n_{i1}) \ne 0$, we have (3).

Formula (3) can be expressed as the flowchart in Fig. 4.

The unnecessary regions of the friction cones have been removed when the disposition H is done. We put forward the following proposition for three-finger equilibrium grasps.

Proposition 3: A necessary and sufficient condition for the three-finger equilibrium grasp is that the intersection of the three double-side friction cones is not empty while the disposition H is done.

The proof of Proposition 3 can be found in [11].

C. The Algorithm

We now attack the problem of determining whether the intersection of the three double-side friction cones is empty or not. There are at most 15 points at which the six boundary lines of the three double-side friction cones intersect. They are points of intersection by two boundary lines of different double-side friction cones except for the three contact points.

Proposition 4: The intersection of the three double-side friction cones is not empty, if and only if any point of intersection by two boundary lines of different double-side cones is not outside of the third double-side friction cone.

According to Proposition 4, we only need to calculate the 12 points of intersect and determine whether one of them lies in a certain double-side friction cone. If there is such a point the grasp is equilibrium, otherwise it is not equilibrium.

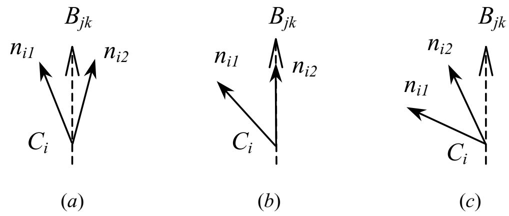


Fig. 5. Point of intersection B_{jk} is: (a) in the interior of the double-side friction cone at C_i. (b) on the border. (c) external.

Let B_{jk} represent one of the points of intersection by the boundary lines of double-side friction cones at C_j and C_k. The positive friction cone at C_i is bounding by n_{i1} and n_{i2} (refer to Fig. 5). If B_{jk} isn't outside of the double-side friction cone at C_i, there must be

$$Sgn(\overrightarrow{C_iB_{jk}} \times n_{i1}) \cdot Sgn(\overrightarrow{C_iB_{jk}} \times n_{i2}) \le 0 \qquad (4)$$

Especially, when B_{jk} is on the border of the double-side friction cone at C_i, there is

$$Sgn(\overrightarrow{C_iB_{jk}} \times n_{i1}) \cdot Sgn(\overrightarrow{C_iB_{jk}} \times n_{i2}) = 0 \qquad (5)$$

According to Proposition 1, a three-finger grasp that achieves nonmarginal equilibrium also achieves force-closure. The intersection of the three double-side friction cone is discrete points or beeline when it is marginal equilibrium grasp. In case of B_{jk} lies strictly inside the double-side friction cone at C_i, any a point inside the infinitesimal neighborhood of B_{jk} belongs to the intersection of the three double-side friction cones, therefore it is nonmarginal equilibrium thus force-closure grasp.

We modify the inequation (4) to (6),

$$Sgn(\overrightarrow{C_i B_{jk}} \times n_{i1}) \cdot Sgn(\overrightarrow{C_i B_{jk}} \times n_{i2}) < 0 \qquad (6)$$

If only a point of intersection satisfying inequation (6), the grasp achieves force-closure.

We summarize the algorithm as follows:

Step 1: processing the disposition H to the friction cones at the three contact points as Fig 4.

Step 2: Calculate the 12 points of intersection by two boundary lines of different double-side friction cones.

Step 3: Calculate the left of (6) for each point. If one of them satisfying (6) the grasp is force-closure, otherwise it isn't.

If all the vertices of the intersection of the three abbreviated double-side friction cones are computed using the above algorithm, an intersection region can be determined. Then we'll provide a measure of how far a grasp is from losing force-closure in the following section.

III. QUANTITATIVE TEST FOR FORCE-CLOSURE GRASPS

From the Proposition 3, there must be an intersection region (more than a point) of the three abbreviated double-side friction cones for a force-closure grasp. Let *S* represent the intersection region and $\alpha_i, i = 1,2,3$ represent the angle of the partial friction cones at three contact points that contribute to *S* (refer to Fig.6). Let $\alpha = \min(\alpha_1, \alpha_2, \alpha_3)$, we take the value of α as the force-closure measure. It means the minimum angle of friction cone that contribute to force-closure.

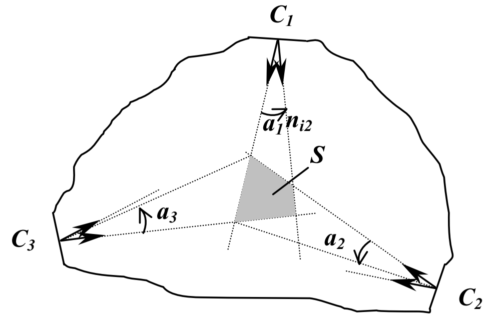
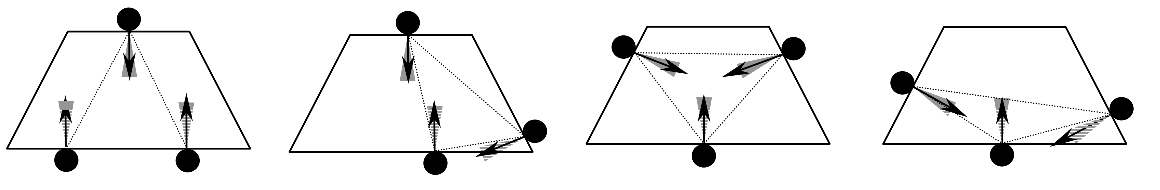


Fig. 6 Partial friction cones contributed to *S*, with *S* shown as a shaded polygonal.

Obviously there is $\alpha = 0$ in the case of non force-closure grasps. Suppose that the minimum angle among the three friction cones is α_0, and then we have $\alpha \in [0, \alpha_0]$.

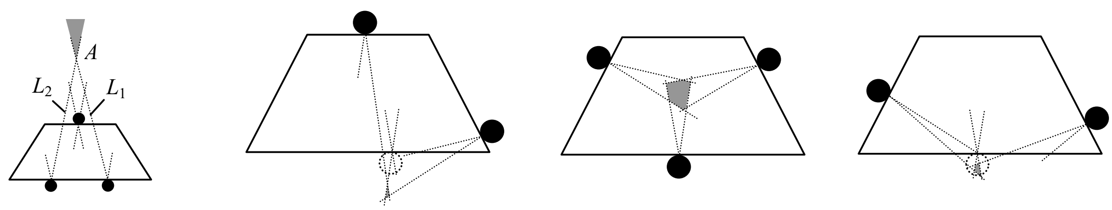
If *S* is a finite region, α can be easily computed by use of the vertices of *S* as shown in Fig.6. Otherwise an approximate algorithm is adopted in which we substitute finite coordinates for infinite ones. For example in the case of Fig.8 (*a*), the intersection region of the three abbreviated double-side friction cones is infinite (part of the region under the quadrilateral isn't shown in the figure). Since that the coordinates of point A are (3.0, 8.5), there are two points (1.0e10, 1.133333e11) and (-1.0e10, -1.133333e11) on the lines L_2 and L_1 respectively. The triangular region with its vertices on those three points approximates to *S*. Then we have $\alpha \approx 20.0$.

In the following section we'll give an example on computing and measuring force-closure grasps of a



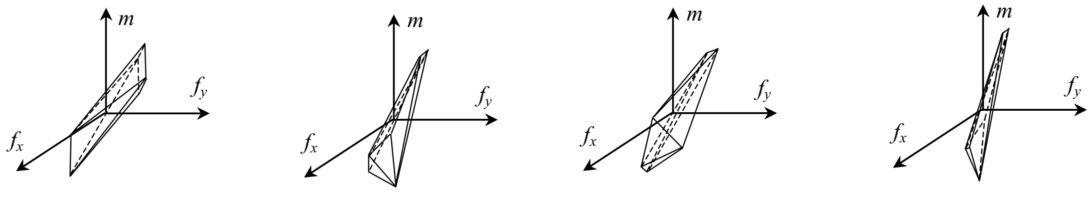
(*a*) (*b*) (*c*) (*d*)

Fig. 7 Force-closure grasps of a quadrilateral (The gray triangle represent the friction cones at the contact points).



(*a*) (*b*) (*c*) (*d*)

Fig. 8 Quantitative test for the force-closure grasps in Fig7 (shade region represents the intersection of the three abbreviated double-side friction cones). The force-closure measure α is (*a*) 20.0, (*b*) 2.3, (*c*) 20.0, (*d*) 3.7.



(a) (b) (c) (d)

Fig. 9 Systems of six wrenches corresponding to the force-closure grasps shown in Fig. 7 respectively, with their convex hull shown as a polyhedron. The radius of a maximal ball centered at the origin and included in the convex hull of the primitive wrenches is (*a*) 0.173, (*b*) 0.023, (*c*) 0.088, (*d*) 0.026.

quadrilateral.

IV. **AN EXAMPLE**

We have computed three-finger force-closure grasps of a quadrilateral using the algorithm advanced in Section 2.3. The friction cone is set to 20º and the sides of the quadrilateral are 3, 3, 3, 6 in length respectively (refer to Fig.7). We choose three contact points on the border of the quadrilateral at random, and compute 10,000 grasps. 1,394 force-closure grasps are found in a total of 273ms of CPU time on a PIII-800 PC in this example. It takes 0.045ms at most in computing a force-closure grasp. The algorithm efficiently reduces the amount of computation required, as compared to linear programming schemes.

There are three types of contact configuration for the force-closure grasps in this example: three contact points lie on the parallelling side of the quadrilateral (Fig. 7(*a*)); two contact points lie on the parallelling side of the quadrilateral respectively and one lies on a bevel side (Fig. 7(b)); three contact points lie on the two bevel sides and bottom side respectively (Fig. 7(*c*)(*d*));

We have computed the force-closure measure α for the grasps as shown in Fig.7. The result is shown in Fig.8, and the measure α is (*a*) 20.0, (*b*) 2.3, (*c*) 20.0, and (*d*) 3.7. As comparing with the measure α, the radius of inner ball in the convex hull of the primitive wrenches is computed as well, we take the scale of moment to force as 1:1 in this example, and the result is shown in Fig. 9.

Because of the non-comparability of forces and moments, the radius of inner ball relates to the scale of moment to force. And there is no such problem in the computing process of measure α. From the result of this example, there seems to exist some difference between the measures of α and the radius of inner ball. For the grasps as shown in Fig.7 (*a*) and (*c*), the measures of the radius of inner ball show that (*a*) is 'better' than (*c*) (0.173:0.088 as shown in Fig.9), but the measures of α show that they are equally good as each other.

V. **CONCLUSION**

The algorithm developed in this paper is quite simple and only needs some algebraic calculations. As comparing to the linear programming schemes, it is suitable to be used in real-time programming.

VI. **ACKNOWLEDGMENTS**

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REFERENCES

[1] J. Hong, G. Lafferiere, B. Mishra, and X. Tan, Fine manipulation with multifinger hands. *IEEE International Conference on Robotics and Automation*, 1990: 1568-1573

[2] X. Markenscoff, L. Ni, C. H. Papadimitriou, The geometry of grasping. *International Journal of Robotics Research*, 1990, 9(1): 61-74

[3] B. Mishra, J. T. Schwartz, and M. Sharir, On the existence and synthesis of multifinger positive grips. *Algorithmica, Special Issue: Robotics*, 1987, 2(4): 541-558

[4] J. K. Salisbury and B. Roth, Kinematic and force analysis of articulated hands. *ASME J. Mechanisms*, Transmissions, Automat. Design, 1982, 105: 33-41

[5] V. D. Nguyen, Constructing force closure grasps, *the International Journal of Robotics Research*, 1988, 7(3): 3–16

[6] J. Ponce and B. Faverjon, On computing three–finger force–closure grasps of polygonal objects, *IEEE Transactions on Robotics and Automation*, 1995, 11(6): 868-881

[7] J. C. Trinkle, A quantitative test for form closure grasps, *IROS*, 1992: 1670-1677

[8] I-M. Chen and J. Burdick, A qualitative test for n-finger force-closure grasps on planar objects with applications to manipulation and finger gaits. *IEEE International Conference on Robotics and Automation*, 1993: 814-820

[9] C. Ferrari and J. F. Canny, Planning optimal grasps, *IEEE Int. Conf. on Robotics and Automation*, 1992: 2290-2295

[10] Y. H. Liu, Computing n-finger form-closure grasps on polygonal objects. *Int. J. Robotics Research*, Vol.19, No.2, pp. 149-158, 2000

[11] J. W. Li, H Liu., and H. G. Cai, On computing three-finger force-closure grasps of 2D and 3D objects. To be published in *IEEE Transactions on Robotics and Automation*

[12] Ji Z. and Roth B., Contact force in grasping and kinematic constraints, in 7th *IFToMM World Congr.*, Sevilla, Spain, 1987: 1219-1222