2D Finite Element Formulation for 3D Temperature Analysis of Layered Hybrid Structures

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Summary:

Three-dimensional (3D) finite element formulations are usually applied for the analysis of temperature fields for hybrid and conventional composite structures. This leads to a high effort with respect to discretisation and computation time, especially for transient calculations.
Using a new formulation basing on a two-dimensional (2D) finite element discretisation it is now possible to approximate the three-dimensional temperature field very effectively.
Through combination of layerwise shape functions in thickness direction (linear or quadratic) and suitable heat transfer conditions at the interfaces the nodal number of degrees of freedom remains independent from the number of layers. In conjunction with shell elements that are generally applied for the stress analysis of thin composite structures it is also easier to transfer temperature data fields due to the same two-dimensional discretisation. This leads to a fast and accurate formulation for the temperature analysis of hybrid composite structures.

Keywords:

Finite elements; thermal analysis; hybrid composite structures; layerwise theory

1 Introduction

One main objective of the thermal analysis is to control if a structure fulfils the thermal requirements and to supply the full three-dimensional temperature distribution as input for the thermo-mechanical analysis. In the case of composites and sandwich structures the layers have different thermal conductivities in different directions. Figure 1 shows different composites and a typical sandwich structure.

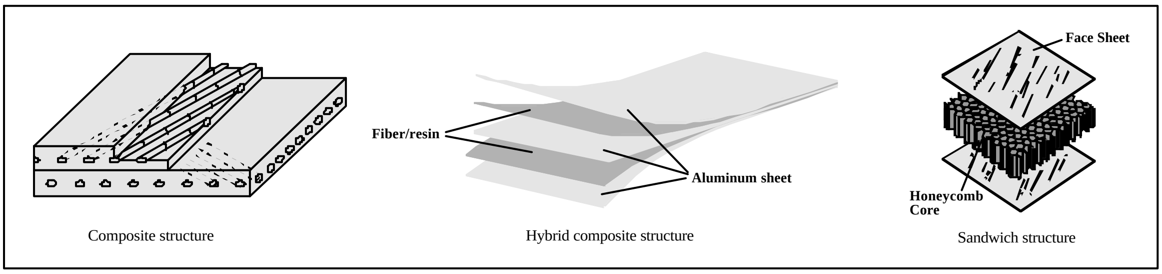

Figure 1: Examples of composite and sandwich structures

For the evaluation of the full three-dimensional temperature distribution 3D finite element or finite difference methods are commonly used. This leads to high modelling and numerical effort, which is not acceptable within a design process. Besides, for thermo-mechanical calculations a two-dimensional model is sufficient, since most commercial finite element codes provide two-dimensional finite elements for composite and sandwich structures. Therefore it is desirable to have finite elements, based on a two-dimensional model, which can calculate the full three-dimensional temperature distribution reducing the modelling and numerical effort drastically. For laminated composites (CFRP) Rolfes [7] has proposed a linear thermal lamination theory which is analogous to the first order shear deformation theory (FSDT). For local effects or transient problems the same author has suggested a quadratic thermal lamination theory. Subsequently, new 2D finite element formulations are outlined which are based on layerwise linear or quadratic temperature distributions in thickness direction. This allows for composite structures with different thermal conductivity in thickness direction for each layer. They can be used especially for lightweight structures in very cold or very hot environments where layers with different thermal and stiffness properties are combined in one lay-up.
It should be mentioned that these finite element formulations facilitate the possibility to be used within a concurrent integrated engineering process. Due to two-dimensional data structure of the thermal model it can be coupled much easier to mechanical models consisting of shell elements than conventional three-dimensional thermal models. This is important for fast and accurate analysis within the preliminary design phase of structural parts.

2 Finite – Element – Formulation

CFRP, hybrid composites and sandwich structures can be idealised as layered structures, see figure 2. For layers in which all modes of heat transfer (heat conduction, radiation and convection) occur (for example honeycomb cores) a thermal homogenisation is necessary. This homogenisation is not a specific requirement for 2D finite elements, but is equally needed if a 3D finite element or finite difference model is applied.
A layerwise discretisation with 3D finite elements is very costly. Therefore different approaches have been made to reduce the modelling effort. For an overview of different methods see [4] and [8]. An useful method for CFRP structures was developed by Rolfes [7]. It describes the thermal lamination theory (TLT), assuming either linear or quadratic temperature distributions over the whole laminate. This theory holds for the following conditions:

- Identical thermal conductivity of all layers in the thickness direction
- No heat-transfer resistance at the interfaces

The linear TLT can then be formulated as

$$T(x,y,z) = T_0^{(b)}(x,y) + z \cdot T_{0,z}^{(b)}(x,y) . \qquad (1)$$

Non-linear temperature distributions in thickness direction can occur in the presence of

- large temperature gradients in the thickness direction in conjunction with temperature-dependent thermo-physical properties
- transient problems with rapid heating
- spatially concentrated thermal loads

In such cases, the quadratic TLT is better suited. It assumes

$$T(x,y,z) = T_0^{(b)}(x,y) + z \cdot T_{0,z}^{(b)}(x,y) + \frac{z^2}{2} T_{0,zz}^{(b)}(x,y)\ . \tag{2}$$

For modelling hybrid structures (e.g. metallic multiwall TPS, hybrid composites (GLARE), sandwiches or hot structures) it is necessary to give up the first condition stated above, and therefore assuming

- different thermal conductivity of each layer in thickness direction.

This leads to the need of layerwise theories.
A linear layered theory (LLT) (conf. Figure 2) was first used by Sipetov [9] for steady state thermal problems. It assumes for each layer *k*

$$T^{(k)}(x,y,z) = T_0^{(k)}(x,y) + z_k \cdot T_{0,z}^{(k)}(x,y) \tag{3}$$

Using two heat transfer equilibrium conditions at each layer interface for the

- temperature,
- heat flux in transverse direction,

the number of functional degrees of freedom can be made independent from the number of layers. This theory was extended to transient problems by Noack and Rolfes [3].
For the same reasons as stated for the TLT, a quadratic layered theory was formulated for transient thermal problems and local heat loads. It reads

$$T^{(k)}(x,y,z) = T_0^{(k)}(x,y) + z_k \cdot T_{0,z}^{(k)}(x,y) + \frac{z_k^2}{2} T_{0,zz}^{(k)}(x,y)\ . \tag{4}$$

By the use of a third heat transfer equilibrium condition at each layer interface for

- the change of the heat flux in transverse direction,

again the number of functional degrees of freedom can be made independent from the number of layers. Based on these theories the finite elements QUADLLT and QUADQLT were developed, showing linear or quadratic temperature distributions in thickness direction for each layer.

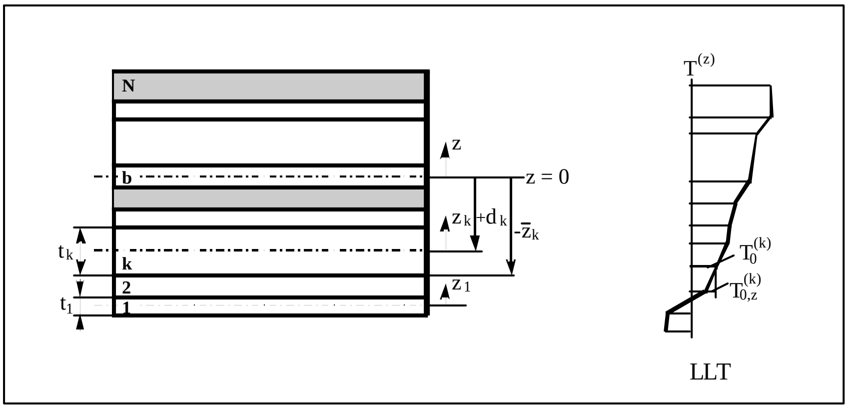

Figure 2: Layered design of composite and sandwich structures

Starting with Fourier's law for layer *k* of anisotropic material

$$\mathbf{q}^{(k)} = -\mathbf{K}^{(k)} \cdot (\operatorname{grad} T)^{(k)} \tag{5}$$

equations (3) and (4) may alternatively be used for the approximation of the temperature *T*. Introducing the above mentioned heat transfer equilibrium conditions at the layer interfaces leads to

$$T^{(k)}(x,y,z) = T_0^{(b)}(x,y) + \tilde{z}_k(z) \cdot T_{0,z}^{(b)}(x,y) \tag{6}$$

for the linear layerwise approximation and

$$T^{(k)}(x,y,z) = T_0^{(b)}(x,y) + \tilde{z}_k(z) \cdot T_{0,z}^{(b)}(x,y) + \tilde{\tilde{z}}_k(z,z^2) \cdot T_{0,zz}^{(b)}(x,y) \tag{7}$$

for the quadratic layerwise approximation. The index *b* denotes the reference layer. For the values of $\tilde{z}_k$ and $\tilde{\tilde{z}}_k$, which are functions of the thickness-coordinate *z*, see [3] and [4]. It is important that for both formulations the functional degrees of freedom remain independent of the number of layers which is crucial for the needed computational effort.
Integrating these equations into the three-dimensional weak formulation for linear steady-state heat transfer

$$\int_\Omega (\operatorname{grad} v)^T \mathbf{K} \operatorname{grad} T \,\mathrm{d}\Omega + \int_\Gamma \mathbf{q}^T \mathbf{n} v \,\mathrm{d}\Gamma = 0 \tag{8}$$

it is possible to split the integration in z-direction from the integration in x- and y-direction. Performing the integration in z-direction analytically, the finite element method has to be applied to the remaining two-dimensional weak formulation

$$\int_A \mathbf{N}^T \tilde{\mathbf{K}} \mathbf{N} J \,\mathrm{d}A + \int_\Gamma \mathbf{q}^T \mathbf{n} v \,\mathrm{d}\Gamma = 0\,. \tag{9}$$

In equation (8) *v* is the test function. The boundary conditions at the edge Γ, considering free convection q_c and heat flux $\overline{q}$, read

$$\mathbf{q}^T \mathbf{n} = q_c + \overline{q}\,. \tag{10}$$

The integration in z-direction leads to the modified heat conduction matrix $\tilde{\mathbf{K}}$. The shape functions and their derivatives are summarised in the matrix $\mathbf{N}$ and the nodal degrees of freedom in the vector J. Equation (9) can now be implemented into an ordinary 2D finite element formulation. For details of the referred values see [3] and [4].

3 Examples

A square plate with a local heat flux of q = 100 kW/m^2 was considered. At the bottom of the plate a convection boundary condition with an ambient temperature of T_∞ = 0 °C and α_c = 30 W/m^{2}K was applied. The geometry is shown in figure 3.

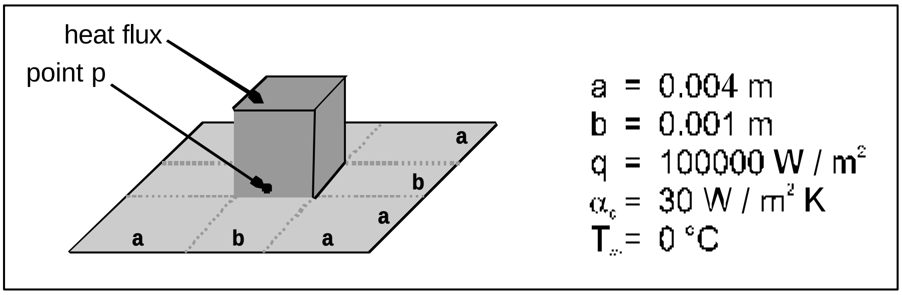

Figure 3: Example problem for the thermal analysis

Two different composites were analysed. The temperature distributions in transverse direction at point P are shown in figure 4. The results show a good agreement between 2D and 3D analysis. Especially, quadratic layered theory and 3D results match excellently. Numerical and modelling effort are drastically reduced by the new finite elements.

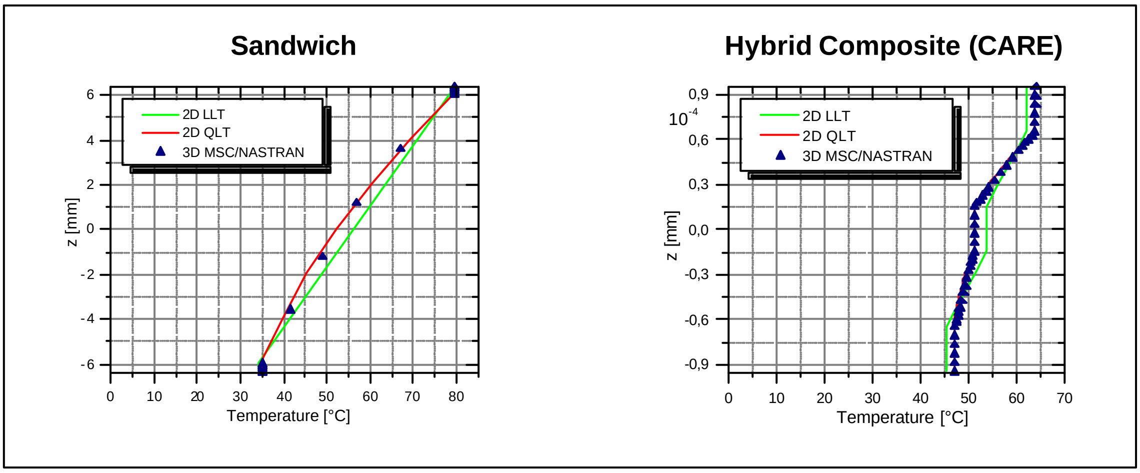

Figure 4: Comparison of 2D and 3D thermal analysis (transverse temperature distribution at point P)

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