HOMOGENIZATION AND TESTS OF A FLAT HOLLOW BODY STRUCTURE

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ABSTRACT: *The Lightex structure is a non homogeneous thinwalled lightweight plate structure which combines different mechanical properties advantageously. For example it can be used for acoustic and thermal isolating also it is able to carry relatively high mechanical loads with respect to its weight. This structure is characterized by one periodic repetitive cubic cell. The aim of this publication is to present a simple homogenization method based on the finite element method. The equivalent stiffness energy of a characteristic cubic cell is computed with finite elements. Stiffness coefficients according to the Kirchhoff plate theory are determined by comparing this energy with the equivalent stiffness energy of a homogeneous plate. The results of this homogenization are stiffness coefficients which can be used to analyze the elastic behavior of thin Lightex plates in large structures.*

1 INTRODUCTION

The research works in the lightweight construction are essentially concentrated on the development of new innovative materials. Some advantages are offered by Lightex structures. It is a cheap and easy to produce plate structure, it is able to carry relatively high mechanical loads with respect to its weight also it provides thermal and acoustic insulation. Lightweight materials are used to increase payloads in vehicle construction. A lightweight container is designed for the combined road and rail transportation. In the course of this project a first range of application is a container front wall. This front wall can be computed by a finite element model. The plate structure necessitated a fine element mesh. Due to the size of an accurate FE-model a amount of memory and computation time is required. Thus the aim is to homogenize the mechanical behavior of the plate structure in order to build a less complicated model of the front wall where a coarser mesh can be used. The homogenization results are stiffness coefficients according to the two dimensional Kirchhoff plate theory. During the initial design phase the local elastic behavior of the structure can be neglected. In order to compute the elastic global deformations the stiffness coefficients are usually sufficient. Thus the global elastic behavior of large Lightex structures can be computed within a short times.

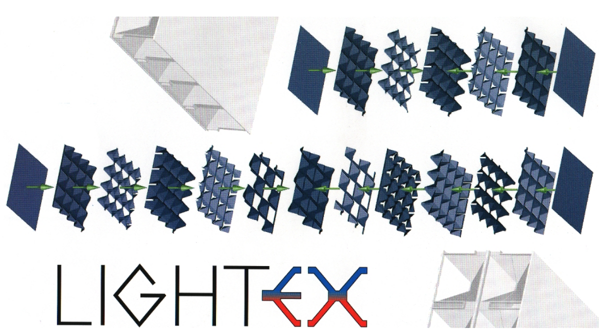

FIG. 1: Lightex structure

The plate structure can be built in wide design variations and with different materials. From there it can be used in many different constructions and applications. This simple homogenization method offers further the possibility to preselect a suitable design and material variation, which is adapted to the considered application.

1.1 THE LIGHTEX STRUCTURE-CONCEPT

The Lightex structure is built by several layers in different directions. 5 layers are combined to build a single plate (figure 2). The five-ply structure consist of 3 different layer types, one intermediate ply and two outer plies, which are shown in the upper part of figure 1. The two outer plies are combined with the intermediate at the top and bottom of the structure. The two bottom outer plies are arranged perpendicular to the top plies which are in the same direction as the intermediate ply. The structure is completed with two plain faces which are used as covers. All layers are adhesively bonded.

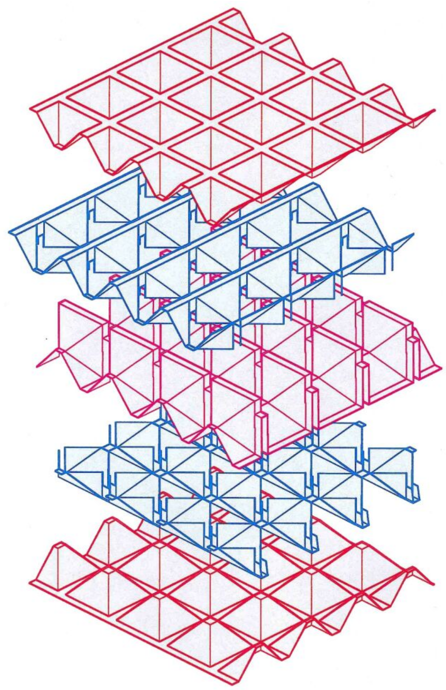

FIG. 2: Five-ply layer model of the Lightex structure

Plates with larger wall thicknesses can be built by stacking one single plate upon another. For this additional plies are necessary. Three combined plies are used in the middle in the structure in order to clamp two five-ply layers together. This structure is shown in the lower part of figure 1. The structure can be made of many different materials; for example plastics, metals or paper. Assumed plastic deformation is not existing, plasticity material properties remain unconsidered. Therefore the finite element model use linear elastic material behavior.

2 MODELLING

2.1 FE-MODEL

The basis of this homogenization method is the symmetry of the structure and the existence of a characteristic cubic cell. A symmetric plate structure is analyzed as shown in lower part of figure 1. The unit-cell - shown in figure 3 - is analyzed by means of a FE-System applying symmetric or anti-symmetric boundary conditions.

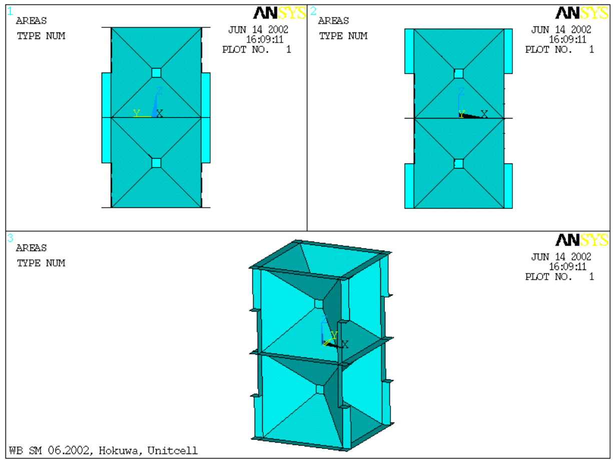

FIG. 3: Finite element model of one unit-cell

2.2 PLATE STIFFNESS COEFFICIENTS

For the design of lightweight structures engineering material properties such as Young's Modulus and stiffness coefficients are significant. During the initial design phase the global elastic behavior of the complete structure is of major concern; local elastic effects within the structure can be neglected. At that stage it is sufficient to describe the global deformation by means of suitable engineering mechanical properties. The Kirchhoff plate theory should suffice for this purpose.

The mechanical behavior of this structure is dependent on the load direction. Due to the cubic and rectangular structure two preferred directions are present. Orthotropic material properties are required. According to the Kirchhoff plate theory orthotropic plate material properties can be considered with

$$\left\{ \begin{array}{c} \mathbf{n} \\ \mathbf{m} \end{array} \right\} = \mathbf{C} \cdot \left\{ \begin{array}{c} \varepsilon \\ \kappa \end{array} \right\} \tag{2-1}$$

whereas $\mathbf{n}$, $\mathbf{m}$ is defined as the internal force and moment vector; ε is the strain and κ is the curvature vector; $\mathbf{C}$ is the stiffness matrix. In the orthotropic principle coordinate system the stiffness matrix

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 & 0 \\ c_{12} & c_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & 0 & c_{45} & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \tag{2-2}$$

is defined by 8 stiffness coefficients. Due to the elastic orthotropic behavior of the structure there are no coupling stiffness coefficients between shell loads and plate deformations. The coupling stiffness coefficients remain unconsidered. The shell and plate stiffness coefficients can be determined independently from each other. In the orthotropic principle coordinate system anti-symmetric deformations, like in-plane shear and torque, are independent from symmetric loads, like tension and bending. Therefore the elastic deformation of the structure can be computed with eight stiffness coefficients c_{ij}.

2.3 EQUIVALENT STIFFNESS ENERGY

The homogenization method is based on the assumption of an equivalent stiffness energy. The elastic energy is calculated by

$$\Pi_i = \frac{1}{2} \oint_V (\sigma^{\mathrm{T}} \varepsilon) dV. \tag{2-3}$$

According to the Kirchhoff theory the tensions in wall direction can be neglected. Therefore the tensions in equation (2-3) could be replaced by the internal forces

$$n_x = \int_{t/2}^{t/2} \sigma_x dz, \quad (2\text{-}4)$$

$$n_y = \int_{t/2}^{t/2} \sigma_y dz, \quad (2\text{-}5)$$

$$n_{xy} = \int_{t/2}^{t/2} \tau_{xy} dz \quad (2\text{-}6)$$

and the internal moments

$$m_x = \int_{t/2}^{t/2} \sigma_x \cdot z dz, \quad (2\text{-}7)$$

$$m_y = \int_{t/2}^{t/2} \sigma_y \cdot z dz, \quad (2\text{-}8)$$

$$m_{xy} = \int_{t/2}^{t/2} \tau_{xy} \cdot z dz \quad (2\text{-}9)$$

by integrating the two dimensional tensions along the wall thickness t. Following, the elastic energy is independent from the plate thickness by a simplification with the internal forces in the form

$$\begin{aligned}\Pi_i = \frac{1}{2} \int_A (&c_{11}\varepsilon_x^2 + c_{22}\varepsilon_y^2 + 2c_{12}\varepsilon_x^2\varepsilon_y^2 + \\ &\ldots + c_{33}\gamma_{xy}^2 + c_{44}\kappa_x^2 + c_{55}\kappa_y^2 + \\ &\ldots + 2c_{45}\kappa_x^2\kappa_y^2 + c_{66}\kappa_{xy}^2) dxdy.\end{aligned} \quad (2\text{-}10)$$

3 Homogenization

The homogenization method is based on the assumption of an equivalent stiffness energy. The basis of this calculation is the symmetry of the structure and the existence of a characteristic repetitive cubic cell. The equivalent stiffness energy of that unit-cell is computed by an FE-model with applied symmetrical or anti-symmetrical boundary conditions which are subjected to the considered load case. The equivalent plate stiffness coefficients are following determined by comparison with the stiffness energy of an homogeneous plate. Four elementary load cases (tension, in-plane shear, bending and torque) are considered in order to calculate eight orthotropic stiffness coefficients. The load cases tension and bending causes symmetrical deformations whereas anti-symmetrical deformations are caused by in-plane shear and torque. Boundary conditions are specified accordingly symmetrical for tension and bending and anti-symmetrical for in-plane shear and torque

3.1 Tension

Tension cause a symmetrical deformation, therefore symmetrical boundary conditions are specified which are shown in figure 4. A uniform displacement u is given. The uniform strain ε_x can be determined simplified with the unit-cell length l_x by

$$\varepsilon_x = \frac{\partial u}{\partial x} \approx \frac{\Delta u}{\Delta x} = \frac{u}{l_x}. \quad (3\text{-}1)$$

At the side where the displacement u is defined the rotation in the y and z directions are prevented. The symmetrical boundary conditions are supplemented at the other sides of the unit-cell as shown in figure 4. In this figure all prevented degrees of freedoms are shown as rotational and translational arrows.

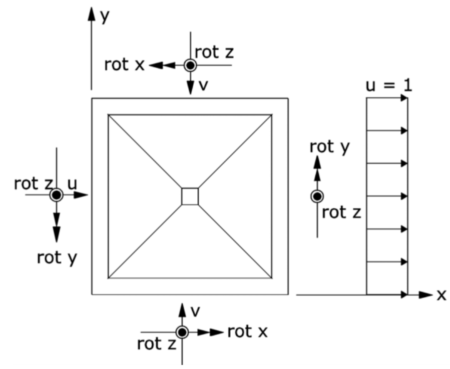

FIG. 4: Boundary conditions for the computation of in-plane tension

The stiffness energy of an homogeneous orthotropic plate is determined by equation (2-10). Due to the boundary conditions only one global tensile deformation is permitted. Therefore the equivalent stiffness energy is calculated with the consideration of the Kirchhoff theory by

$$\Pi_{i,11} = \frac{1}{2}\int_A c_{11}\varepsilon_x^2 dA. \tag{3-2}$$

This equivalent stiffness energy is computed with the FE-model. The cross sectional area A_x is determined simplified by the height and wide of the unit-cell. Following the equivalent stiffness coefficient c_{11} can be determined as

$$c_{11} = \frac{2\cdot\Pi_{i,11}}{A_x\cdot\varepsilon_x^2} \tag{3-3}$$

Basically the identical procedure is used to determine the equivalent stiffness coefficient c_{22}, except that a uniform node displacement v in y-direction is given.

Adapted boundary conditions are necessary, in order to compute the transverse stiffness coefficient c_{12}. The boundary conditions are shown in figure 5. The displacement v in transverse direction is determined by a set of coupled node displacements. The nodes at the y-side of the unit-cell are coupled in y-direction therefore a uniform node displacements v results. The uniform resulting displacement of the coupled node set is used to calculate a transverse strain

$$\varepsilon_y = \frac{\partial v}{\partial y} \approx \frac{\Delta v}{\Delta y} = \frac{v}{l_y}. \tag{3-4}$$

The equivalent transverse stiffness coefficient c_{12} is calculated by the tensile stiffness coefficients c_{11}, c_{22} and the equivalent stiffness energy of a homogeneous orthotropic plate (2-10). Two strains are required; the given strain ε_x as shown in figure 5 and a strain ε_y, which results from the FE computation. The transverse stiffness coefficient is calculated by

$$c_{12} = \frac{\Pi_{i,12}}{A_x\cdot\varepsilon_x\varepsilon_x} - \frac{\varepsilon_x}{2\cdot\varepsilon_y}c_{11} - \frac{\varepsilon_y}{2\cdot\varepsilon_x}c_{22} \tag{3-5}$$

3.2 IN-PLANE SHEAR

According to the stiffness matrix (2-2) there are no stiffness coefficients between shear loads and tensile deformations. Therefore a shear loading in the orthotropic principle coordinate system leads to a shear distortion without tensile deformation. A anti-symmetric strain state is caused by in-plane shear loads. Anti-symmetric boundary conditions are specified, in order to determine the equivalent in-plane shear stiffness coefficient c_{33}. The boundary conditions are shown in figure 5. The unit-cell is deformed by uniform displacements. The shear

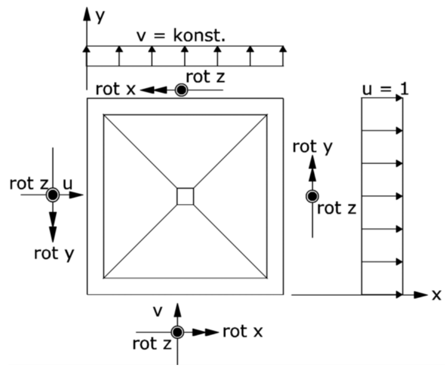

FIG. 5: Boundary conditions for the computation of transverse stiffness coefficient

strain can be simplified determined with the length of the unit-cell l_x, l_y and the displacement u, v by

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \approx \frac{\Delta u}{\Delta y} + \frac{\Delta v}{\Delta x} = \frac{u}{l_y} + \frac{v}{l_x}. \tag{3-6}$$

Due to the boundary conditions there are no global tensile strains and curvatures. The equivalent shear stiffness coefficient of a homogeneous plate is computed by means of the stiffness energy (equation(2-10)) with

$$c_{33} = \frac{2 \cdot \Pi_{i,33}}{\gamma_{xy}^2 \cdot A_x}. \tag{3-7}$$

The cross sectional area A_x can be determined by the height and wide of the unit-cell.

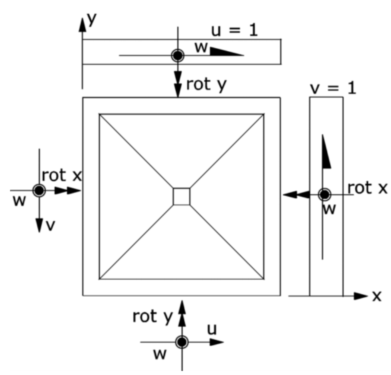

FIG. 6: Boundary conditions for the computation of in-plane shear

3.3 BENDING

Due to the elastic orthotropic behavior there are no coupling effects between plate loads and shell deformations. Therefore the coupling stiffness coefficients in the plate stiffness matrix (2-2) are neglected. Bending is causing a symmetrical deformation. Therefore symmetrical boundary conditions are specified as shown in figure 7. A uniform curvature is necessary for the determination of the bending stiffness coefficient. In order to determine a uniform curvature, the node displacements u are coupled with a rotation φ_y along the y-axis in the midplane of the unit-cell by a couple node set in the form

$$u = \varphi_y \cdot z. \tag{3-8}$$

The largest displacements are achieved by the nodes on the plate surfaces. The curvature

$$\kappa_x = \frac{\partial \varphi_y}{\partial x} \approx \frac{\Delta \varphi_y}{\Delta x} = \frac{\varphi_y}{l_x} \tag{3-9}$$

can be simplified determined by the length l_x of the unit-cell. The equivalent stiffness energy of a homogeneous plate is used to determine the equivalent bending stiffness coefficient. Due to the boundary conditions of the FE-model there is only the curvature along the x-axis. Therefore the stiffness energy of a homogeneous orthotropic plate is only affected by the curvature κ_x. The equation (2-10) is solved by a consideration the bending stiffness coefficient

$$c_{44} = \frac{2 \cdot \Pi_{i,44}}{A_x \cdot \kappa_x^2}. \tag{3-10}$$

The equivalent stiffness energy is computed by the FE-model. The cross sectional area A_x can be determined simplified by the height and weight of the unit-cell as bevor. The identical procedure is used to determine the equivalent bending stiffness coefficient c_{55} by defining a curvature κ_y in the y-direction.

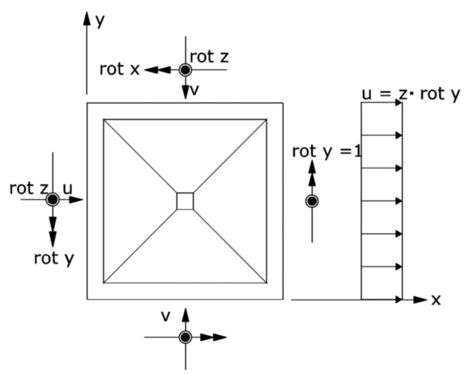

FIG. 7: Boundary conditions for the computation of bending stiffness coefficient

Two curvatures are required, in order to determine the transverse stiffness coefficient c_{45}. At the x-side a uniform curvature κ_x is given by a set of couple node displacements and a uniform rotation φ_y similarly to the given curvature for the determination of the bending stiffness coefficient (equation (3-8)). The transverse curvature κ_y is basically defined identical except that this curvature is defined in the transverse direction and the rotation φ_x is computed by the FE-model. The transverse curvature

$$\kappa_y = \frac{\partial \varphi_x}{\partial y} \approx \frac{\Delta \varphi_x}{\Delta y} = \frac{\varphi_x}{l_y} \tag{3-11}$$

is computed by the FE-model. Symmetrical boundary conditions are used which are shown in figure 8.

Two curvatures must be considered for the calculating of the equivalent stiffness energy of a homogeneous plate. The equivalent transverse bending stiffness coefficient is determined by

$$c_{45} = \frac{\Pi_{i,45}}{\kappa_x \kappa_y A_x} - \frac{\kappa_x}{2 \cdot \kappa_y} c_{44} - \frac{\kappa_y}{2 \cdot \kappa_x} c_{55}. \tag{3-12}$$

Following, the equivalent transverse stiffness coefficient can be determined with the equivalent stiffness energy $\Pi_{i,45}$ of the FE-model and the transverse curvature κ_y.

3.4 TORQUE

Torsional deformations are the result of an anti-symmetrical load case. A torsional curvature κ_{xy} is required, in order to determine the torsional stiffness coefficient c_{66}. This curvature is achieved by two rotations. Translational displacements are coupled with these rotations around the side normal directions in the form

$$u_i = \varphi_j \cdot u_k \tag{3-13}$$

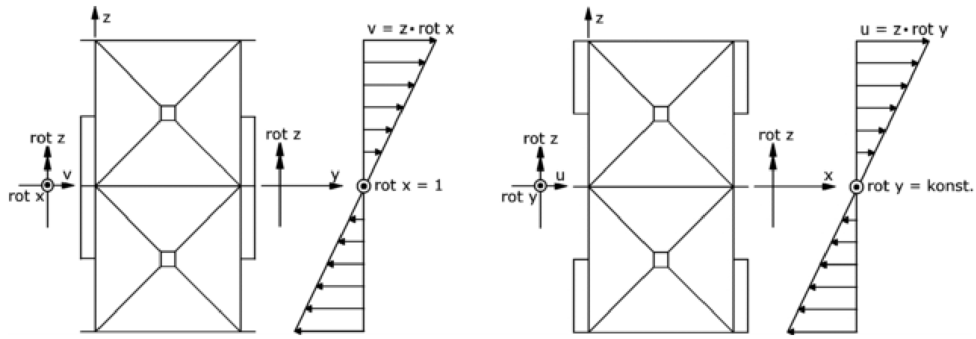

FIG. 8: Boundary conditions for the computation of the transverse bending stiffness coefficient

According to figure 9 the node displacements v are coupled with a uniform rotations φ_x at the midplane of the unit-cell. At the positive y-side a uniform rotation φ_y is defined and coupled with node displacements u. Anti-symmetrical boundary conditions are used as shown in figure 9. A torsional curvature κ_{xy} can be determined simplified with the rotation φ_x and the unit-cell length l_x, l_y. The torsional curvature is determined by

$$\kappa_{xy} = \frac{\partial \varphi_y}{\partial x} = \frac{\partial \varphi_x}{\partial y} \approx \frac{\Delta \varphi_y}{\Delta x} = \frac{\varphi_y}{l_x}. \tag{3-14}$$

The torsional stiffness coefficient is determined by means of the equivalent stiffness energy of a homogeneous plate by

$$c_{66} = \frac{2 \cdot \Pi_{i,66}}{\kappa_{xy}^2 \cdot A_x}. \tag{3-15}$$

The area A is considering the height and weight of the unit-cell.

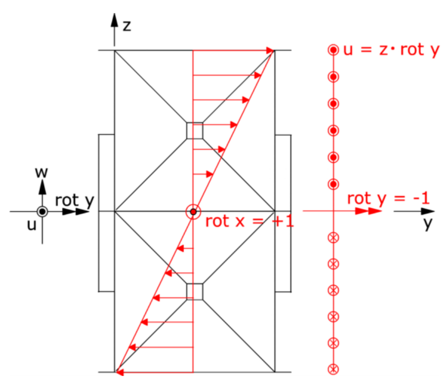

FIG. 9: Boundary conditions for the computation of the torsional deformation

4 NUMERICAL RESULTS

The aim of this homogenization method is to determine stiffness coefficients which can be applied for finite element simulations or analytical calculations. Often used engineering material properties are the Young's modulus and the transverse contraction. These properties can be calculated by the shell stiffness coefficients and the height t of the unit-cell by

$$E_x = \frac{c_{11}}{(1-\nu_x \nu_y)t}, \tag{4-1}$$

$$E_y = \frac{c_{22}}{(1-\nu_x \nu_y)t}, \tag{4-2}$$

$$G_{xy} = c_{33}/t. \tag{4-3}$$

The transverse contraction is determined by

$$\nu_x = \frac{c_{12}}{c_{11}}, \tag{4-4}$$

$$\nu_y = \frac{c_{12}}{c_{22}}. \tag{4-5}$$

The elastic behavior of large structures can be computed by the use of finite elements. The stiffness coefficients, which are required for this computations are determined by a comparison of the elastic energy. Due to the elastic orthotropic behavior of the structure the plate stiffness coefficients must be dependent on the load direction. These direction dependance can be calculated by a matrix transformation

$$\overline{\mathbf{C}} = \mathbf{T}^{\mathrm{T}} \cdot \mathbf{C} \cdot \mathbf{T} \tag{4-6}$$

with the transformation matrix

$$\mathbf{T} = \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & \frac{1}{2}\sin 2\alpha \\ \sin^2\alpha & \cos^2\alpha & \frac{1}{2}\cos 2\alpha \\ -\sin 2\alpha & \sin 2\alpha & \cos 2\alpha \end{bmatrix}, \tag{4-7}$$

which is given by a transformation angle α. Due to the orthotropic material behavior there are no stiffness coefficients between plate loads and shell deformations. The stiffness coefficients between anti-symmetric loads and symmetric deformations c_{13}, c_{23} can be calculated by equation (4-6). The polar-plot 10 shows the magnitude und direction of the shell stiffness coefficients. The tensile shell stiffness coefficients are quasi isotropic. The stiffness coefficients between in-plane shear loads and symmetric deformations are zero in the orthotropic principle axis but varying in the other directions.

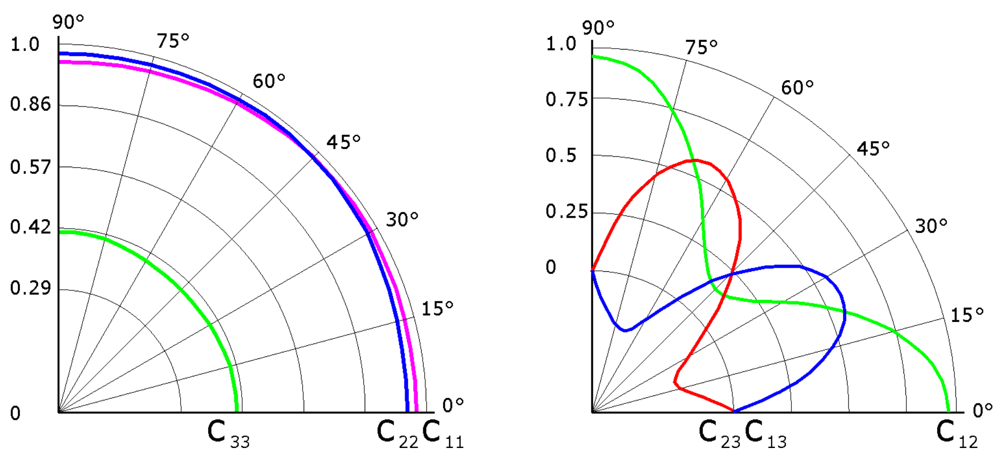

FIG. 10: Polar plots of the tensile stiffness coefficients

5 CONCLUSION

The Kirchhoff plate theory has been approved for the design of large thinwalled lightweight structures. The aim of this publication is to present a simple homogenization method, which offers the possibility to determine stiffness coefficient according to the Kirchhoff theory. The global elastic behavior of large structures can be calculated with these material properties. During the initial design phase engineering material properties, e.g. the E-Modulus and the transverse contraction, are of major concern. This properties can be used to preselect a suitable structure and material which are adapted to the considered application and load case.

LIST OF SYMBOLS

A	cross sectional area
$\mathbf{C}$	stiffness matrix
c_{ij}	stiffness coefficients
E_x, E_y	elastic Modulus
u, v, w	displacements
x, y, z	coordinates
$\mathbf{T}$	transformation matrix
ε, ε_{ij}	strain, strain-vector
κ, κ_{ij}	curvature
σ, σ_{ij}	stress
τ, τ_{ij}	shear stress
Π_i, $\Pi_{i,ij}$	stiffness energy
φ, φ_{ij}	rotation
ν_x, ν_y	transverse contraction

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