TRANSPORTPROZESSE &

ERSTARRUNG

- **Transportprozesse**
  - Diffusion
  - Soret-Effekt
  - Wärmeleitung
- **Erstarrung**
  - Diffusives Wachstum
  - Konstitutionelle Unterkühlung
  - Thermosolutale Konvektion

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DIFFUSION-THEORIE

Thermisch: $j_T = - k \; \nabla T$ (Fourier)

Solutal: $j_c = - D \; \nabla c$ (Fick)

Kontinuitätsgleichung:

$$\frac{\partial c}{\partial t} + \nabla j = 0$$

Diffusionsgleichung:

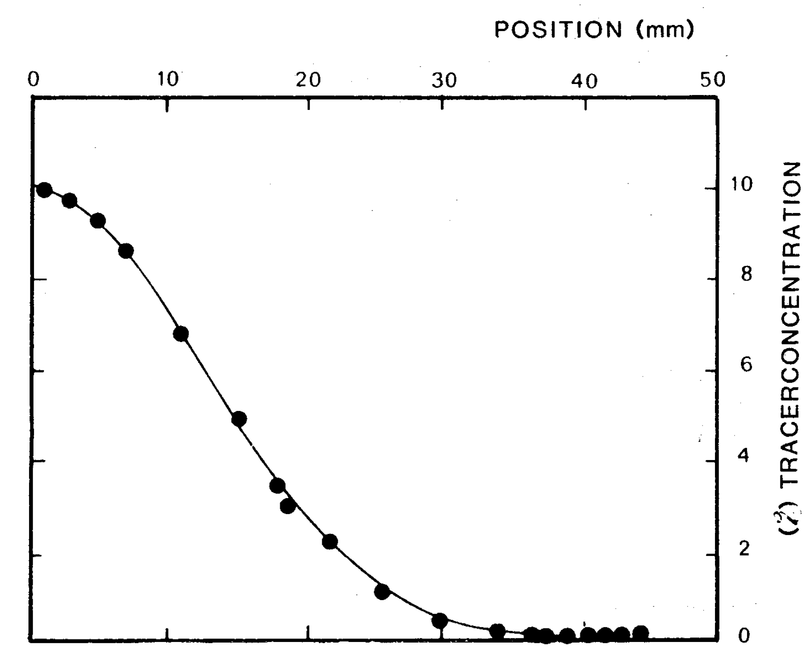
$$\frac{\partial c}{\partial t} = \nabla^2 c$$

1-dimensionale Lösung:

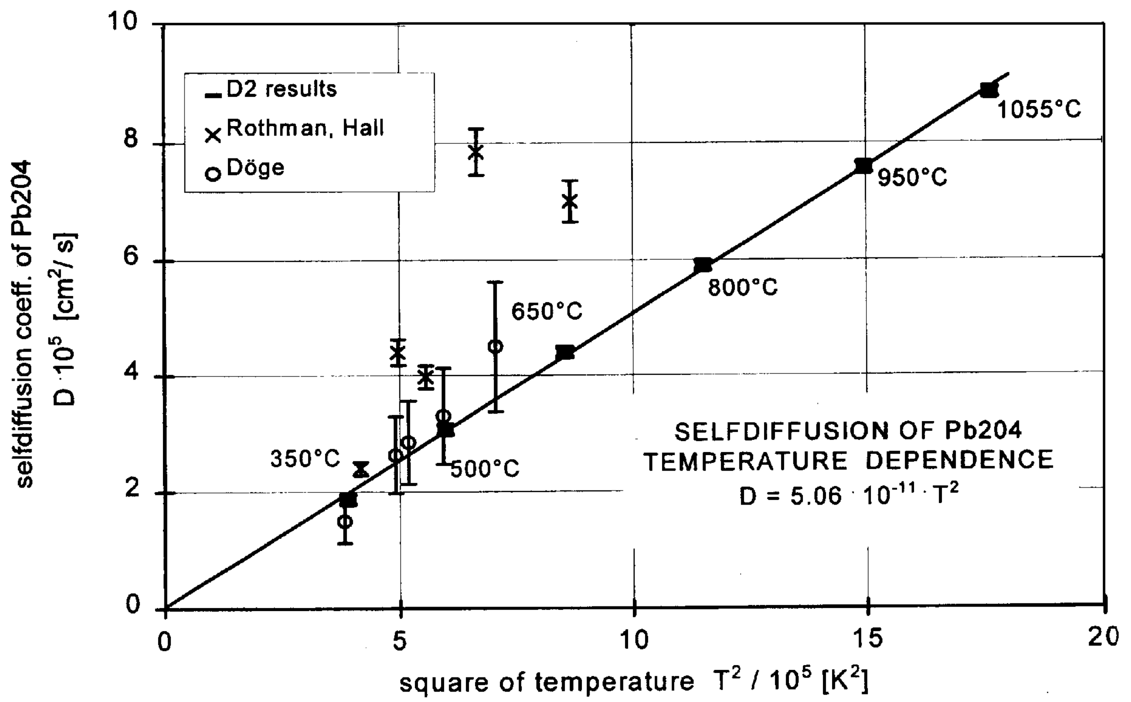
$$c(x,t) = \frac{c_0 d}{\sqrt{\pi D t}} \exp\left(-\frac{x^2}{4Dt} \right)$$

(Wärmetransport analog)

DIFFUSION-EXPERIMENT

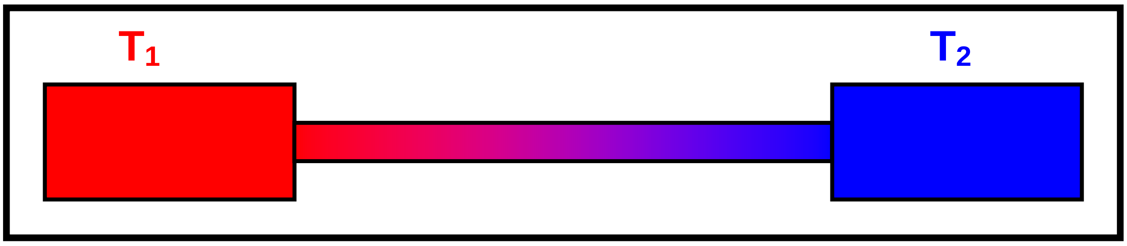


Concentration profile of Sn 112 (810°C, capillary 1 mm Ø):
Space Experiment (Frohberg, Kraatz, Wever)

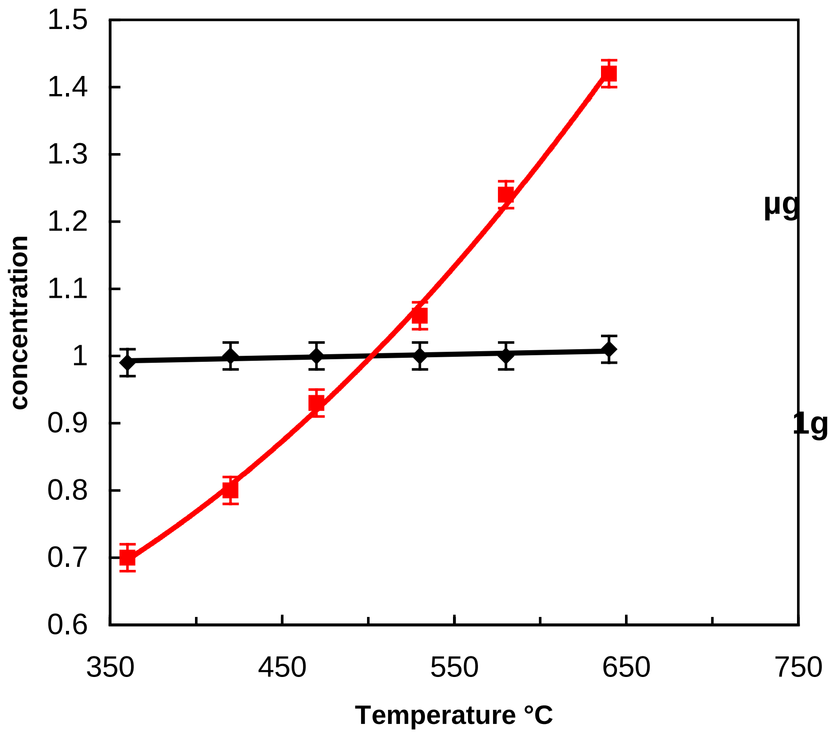


Soret-Effekt

$$\frac{\partial c}{\partial t} = \nabla\left(D\nabla c + S\nabla T\right)$$



Soret effect Co in Sn, T_0 = 500 °C

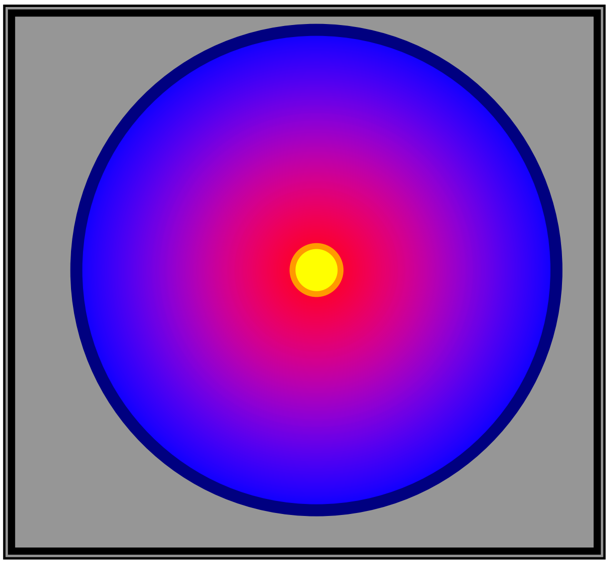


$$\nabla c = -\frac{S}{D}\nabla T$$

WÄRMELEITUNG

$$\rho c_p \frac{\partial T}{\partial t} = k \nabla^2 T + \rho \dot{Q}_{ext}$$

Transient Hot Wire in Zylindergeometrie:

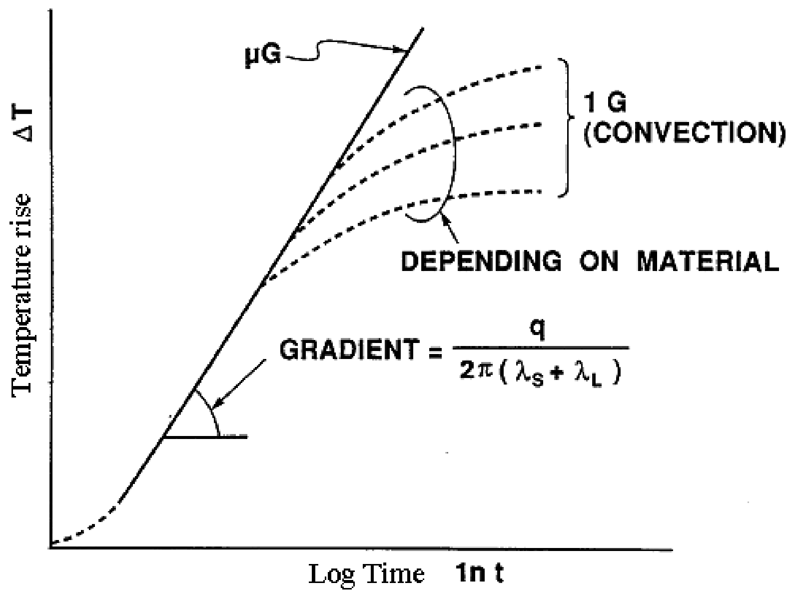


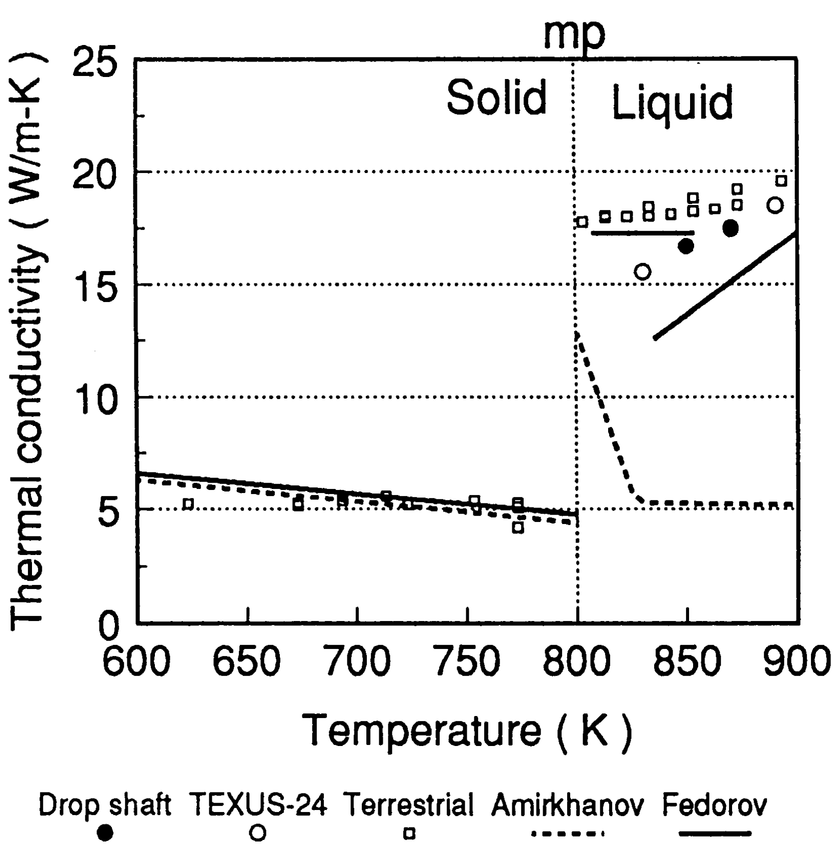
Randbedingung: $\dot{\mathrm{Q}}_{\mathrm{ext}}(r,t) = \dot{q}\,\delta(r)\theta(t)$

Asymptotische Lösung:

$$\Delta T(R_0,t) = -\frac{\dot{q}}{4\pi k}\,\mathrm{Ei}\left(-\frac{R_0^2}{4kt}\right) \approx \frac{\dot{q}}{4\pi k}\ln t$$

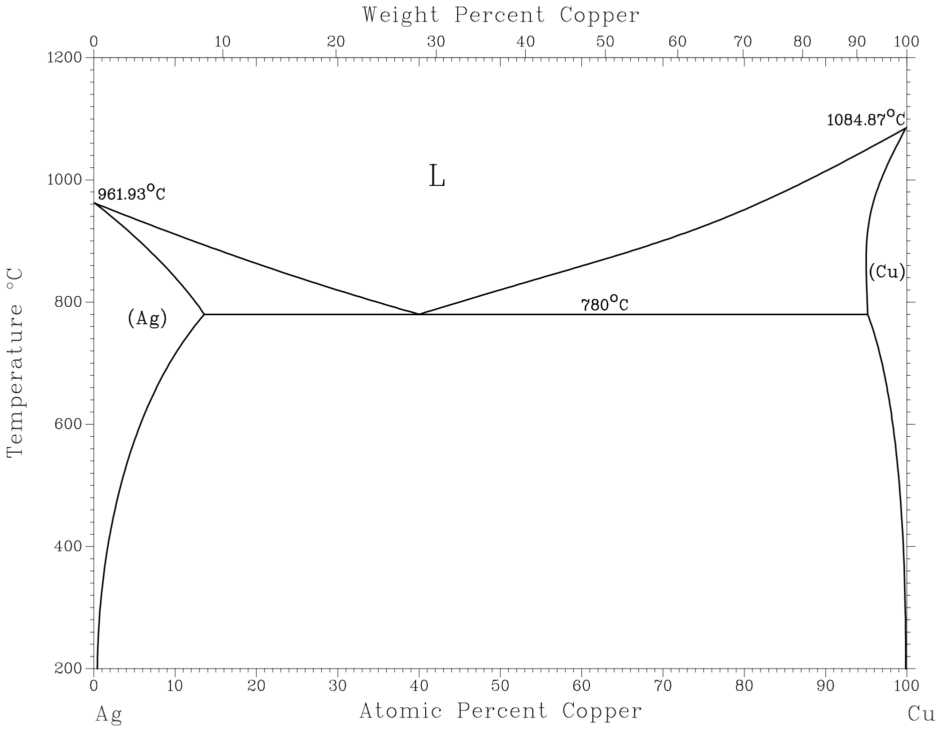
WÄRMELEITUNG





Erstarrung (1)

Typisches Phasendiagramm (Ag-Cu):



Verteilungskoeffizient: $k = \frac{c_S}{c_L} < 1$

Konzentrationsüberschuss:

$$\Delta c = c_L - c_S = (1-k)c_L$$

Erstarrung (2)

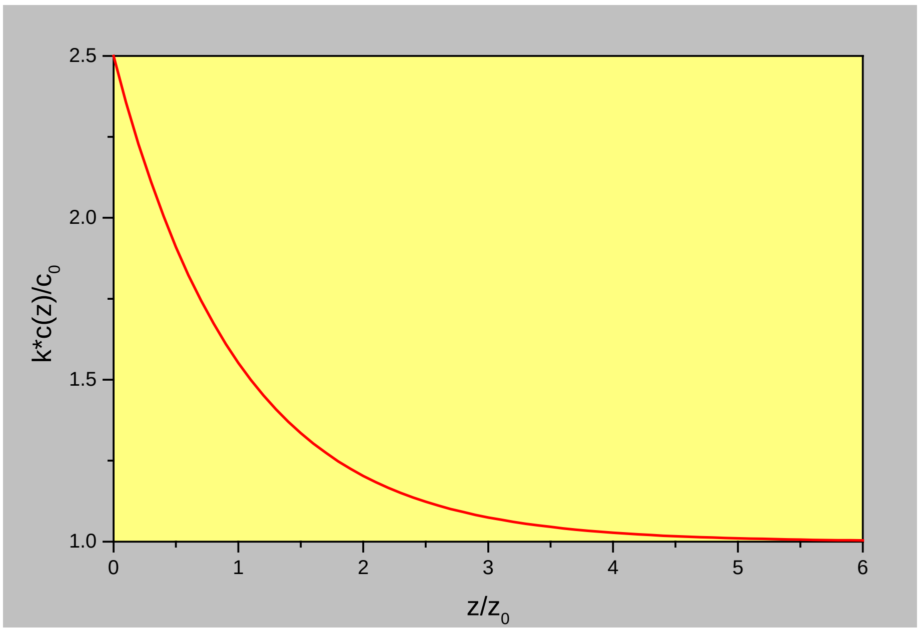
Stationäres Wachstum: $\frac{\partial c}{\partial t} = 0$

$$v_0 \nabla c = D \nabla^2 c$$

1-dimensionale Lösung:

$$c(z) = c_0 \left(1 + \frac{1-k}{k} \exp\left[-\frac{v_0}{D} z \right] \right)$$

$$c(0_+) = c_0/k, \quad c(0_-) = kc(0_+) = c_0$$



KONSTITUTIONELLE UNTERKÜHLUNG

Gerichtete Erstarrung: $T(z) = T_0 + Gz$

Liquidustemperatur: $T_L = T_L(c) \approx T_m - mc$

$c=c(z) \rightarrow T_L = T_L(z)$

Konstitutionelle Unterkühlung:

$$\boldsymbol{T(z_0) < T_L(z_0) \Leftrightarrow G < T_L'(0)}$$

$$T_L'(0) = \left.\frac{dT_L}{dz}\right|_{z=0} = \frac{mc_0(1-k)v_0}{kD}$$

Rayleigh-Konvektion:

$$G > \frac{\kappa\eta\, Ra^c}{\rho\beta L^4 g}$$ $\Rightarrow$ **Mikrogravitation !**

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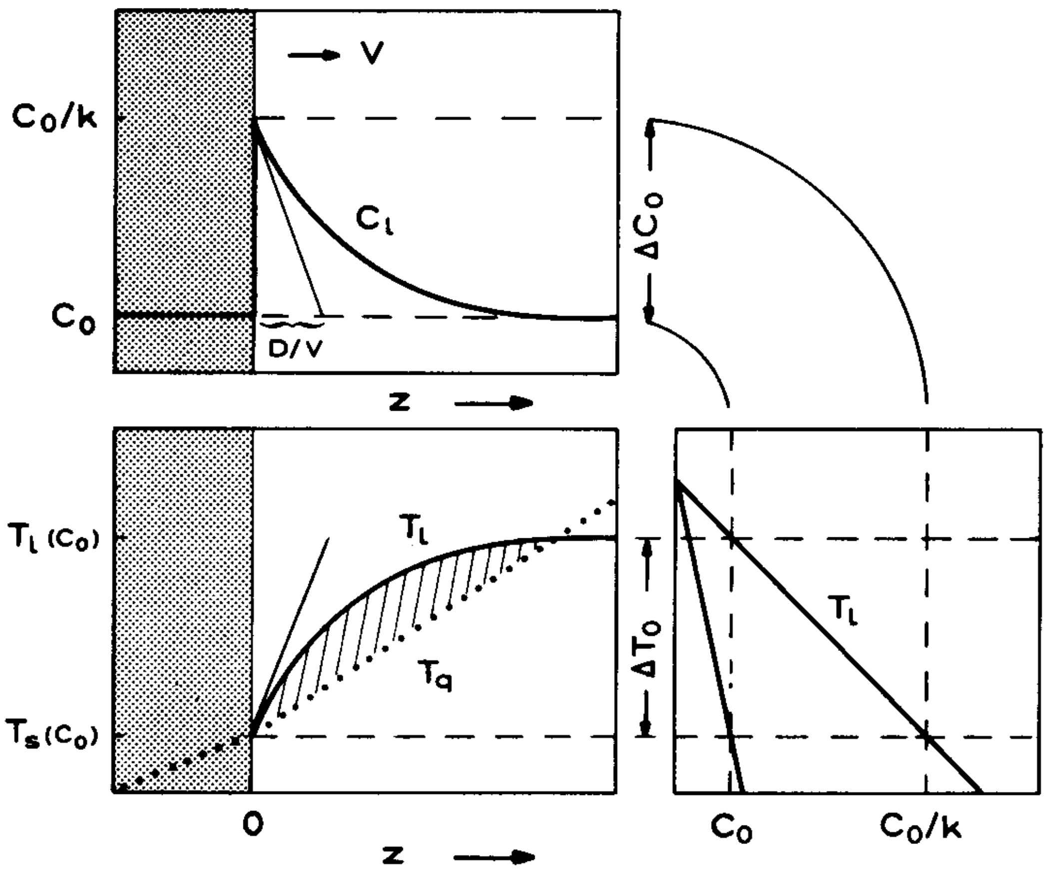

THERMOSOLUTALE KONVEKTION

1-dimensionale Betrachtung:

$$\frac{d\rho(c,T)}{dz} = \frac{\partial\rho}{\partial c}\frac{\partial c}{\partial z} + \frac{\partial\rho}{\partial T}\frac{\partial T}{\partial z}$$

$$\frac{\partial c}{\partial z} = -\frac{(1-k)v_0}{kD}\exp\left(-\frac{v_0 z}{D}\right) < 0, \qquad \frac{\partial T}{\partial z} = G > 0$$

$$\frac{\partial\rho}{\partial c} = \alpha\rho, \qquad \frac{\partial\rho}{\partial T} = -\beta\rho\ , \quad \beta > 0$$

$\Rightarrow$ Vorzeichenwechsel in $d\rho/dz$, falls

$$-\alpha\frac{(1-k)v_0}{kD} > \beta G$$, d.h. $\alpha < 0$

$\Rightarrow$ keine stabile Schichtung möglich

$\Rightarrow$ Rayleigh-Konvektion für $Ra > Ra^c$

$\Rightarrow$ **Mikrogravitation !**

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