Large Angle Maneuver of an Underactuated Small Satellite Using Two Wheels

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Abstract
Failure of mechanical controllers onboard a satellite is a well-known phenomenon that has already been disastrous for several space missions. In the case studied here, we take as an example the mini-satellite UoSat-12 built by SSTL at University of Surrey. The Z-axis reaction wheel of this satellite has failed and the 3-axis control performance using magnetorquing is being very limited. The use of thrusters would involve undesired fuel consumption. As an alternative, we present here some latest theory, which shows how full 3-axis control can still be achieved, using the two remaining reaction wheels from a standard orthogonal 3-wheel configuration. Using a novel nonlinear time invariant and discontinuous approach, we show that the attitude is precisely and rapidly altered, without transient oscillations, to the required earth pointing. The case of a small non-zero total momentum is illustrated. One consequence of these results is that a fully redundant 3-axis control can be envisaged using a 3-wheel configuration.

1. Introduction
It is now well known that only non-smooth control laws can be developed to stabilize nonholonomic systems, including underactuated satellites [2].
In this paper, we focus on the case of a satellite controlled using only two reaction wheels in the event of one wheel failure. Using two wheels, the only 3-axis stabilizing control law in the literature is a time varying control law that ensures the stabilization for a zero total momentum satellite, but the system goes through important transient oscillations [1].
In this paper, we show how a discontinuous control approach can be used to achieve stability and large angle maneuvers for underactuated satellites.

Notations:
$A^I_B$: Direction cosine matrix from inertial reference to body frame
$I = [I_1, I_2, I_3]^T$: Inertia tensor of the body of the satellite about its centre of mass.
\( \mathbf{P} = [p_1, p_2, p_3]^T \): Attitude of the satellite using Rodriguez parameters.

\( ? = [\omega_1, \omega_2, \omega_3]^T \): Vector of the angular velocity in body fixed reference frame.

\( \mathbf{I}_{ii} \): Momentum of inertia of the \( i^{th} \) wheel.

\( \alpha_i, z_i \): Rotation angle, and unit vector of the \( i^{th} \) wheel rotational axis, respectively.

\( \mathbf{h} = [h_1, h_2, h_3]^T \): Angular momentum generated by the wheels in the body frame.

\( \mathbf{H}, \mathbf{L} \): Total angular momentum in the inertial and body frames respectively.

2. DYNAMIC MODEL

If no external disturbance torque is assumed, we have:

\[
\mathbf{L} + ? \times \mathbf{L} = 0
\] (1)

In the case of a reaction wheel failure on the Z-axis ( \( N_3 = h_3 = 0 \) ), the dynamic model is:

\[
\begin{align*}
I_1 \dot{\omega}_1 &= (I_2 - I_3)\omega_2 \omega_3 + N_1 + \omega_1 h_2 \\
I_2 \dot{\omega}_2 &= (I_3 - I_1)\omega_3 \omega_1 + N_2 - \omega_3 h_1 \\
I_3 \dot{\omega}_3 &= (I_1 - I_2)\omega_1 \omega_2 - \omega_1 h_2 + \omega_2 h_1
\end{align*}
\] (2)

Where \( N_i = -\frac{dh_i}{dt} \) is the internal torque applied by the wheels to the satellite.

3. KINEMATIC MODEL

Rodriguez parameters are used here for the control. These parameters are derived from the standard Euler axis/angle representation

\[
\mathbf{P} = [p_1, p_2, p_3]^T = \tan(\phi/2) . \mathbf{e}
\] (3)

where \( \mathbf{e} \) is the Euler axis and \( \phi \) is the rotation about it. The kinematic equation of a rigid body (satellite) using Rodriguez parameters is:

\[
\begin{align*}
\dot{p}_1 &= \frac{1}{2} (\omega_1 - (p_3 - p_1 p_2) \omega_2 + (p_2 + p_1 p_3) \omega_3 + p_1^2 \omega_1) \\
\dot{p}_2 &= \frac{1}{2} (\omega_2 + (p_3 + p_1 p_2) \omega_1 - (p_1 - p_2 p_3) \omega_3 + p_2^2 \omega_2) \\
\dot{p}_3 &= \frac{1}{2} (\omega_3 - (p_2 - p_1 p_3) \omega_1 + (p_1 + p_2 p_3) \omega_2 + p_3^2 \omega_3)
\end{align*}
\] (4)

We first assume a zero total angular momentum satellite

It can be shown that the kinematic model of the zero total momentum satellite, \( \mathbf{H} = 0 \), simply reduces to the well-known Brockett integrator [1]:

\[
\begin{align*}
\dot{p}_1 &= u_1 \\
\dot{p}_2 &= u_2 \\
\dot{p}_3 &= -p_2 u_1 + p_1 u_2
\end{align*}
\] (5)

Where \( u_1, u_2 \) are redefined control inputs that depend on the control torques.
4 TIME INARIANT DISCONTINUOUS CONTROLLER

Nonlinear discontinuous control has already been proposed in order to stabilize nonlinear and nonholonomic systems [3], and in the case of underactuated satellites using pairs of thrusters [2]. Our parameterization of the problem here has to be different. For our two reactions wheels control system, we want to design a controller that first stabilizes the unactuated Z-axis.

\[ \dot{p}_3 = -p_2u_1 + p_1u_2 = -g p_3 \]  

(7)

where \( g \) is a positive constant of the controller.

The relation (8) is ensured using the control law:

\[ u_1 = -kp_1 + g \frac{p_2}{p_1 + p_2} p_3 \]
\[ u_2 = -kp_2 - g \frac{p_1}{p_1 + p_2} p_3 \]

(8)

where \( k \) is another positive constant of the controller.

The asymptotic stability of the variable \( p_3 \) is trivial. We can also demonstrate stability of \( p_1, p_2 \). We consider a variable: \( V = p_1^2 + p_2^2 \), simple calculations yield:

\[ \dot{V} = -kV \]

(9)

Therefore, the manifold \( D = \{(p_1, p_2, p_3) \in \mathbb{R}^3 \mid p_1^2 + p_2^2 = 0\} \), is also exponentially attractive and stability is consequently ensured for the complete attitude to zero.

5. NUMERICAL SIMULATIONS

The parameters of the mini-satellite UoSAT-12 are used for the simulation are:

\[ I_1 = 40.45 \text{ kg.m}^2, I_2 = 42.09 \text{ kg.m}^2, I_3 = 40.36 \text{ kg.m}^2 \]
\[ I_{w1} = 8.10^{-3} \text{ kg.m}^2, I_{w2} = 7.710^{-3} \text{ kg.m}^2 \]

The controller parameters were determined empirically to be best at: \( g = 0.01, k = 0.004 \).

Initial attitude condition: \( p_1(0) = -1, p_2(0) = -1 \) and \( p_3(0) = -1 \), which represents the state of an ‘upside down’ UoSat-12.

In the zero total momentum case, we have stabilization with good performance in Fig(1). In Fig(2), we obtain constant amplitude residual oscillations for a small nonzero momentum (The momentum can be made small initially via detumbling).

6. CONCLUSION

Our work shows that nonlinear discontinuous control theory can be highly effective in achieving attitude control of an underactuated satellite using two wheels. Fast and decisive slewing in all 3 axes is achievable without transient oscillations.
Fig (1): Euler angles and control torque of UoSAT12 using the discontinuous controller. (Zero momentum mode)

Fig (2): Euler angles and control torque of UoSAT12 using the discontinuous controller. (Satellite with small momentum $H = H_0 = 0.01 I_{3×3}$)

7. REFERENCES