Abstract for the Onera NXO method

Case 1.6 Vortex Transport by uniform flow.

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1/ Code description

Discretization by cell-centered Finite Volume Method,

Upwind-biased convective scheme based on characteristic splitting of the conservative variables at the cell interface, no added artificial dissipation operators,

Evaluation of left and right conservative variables as surface averages on the interface interpolated from the volume averaged conservative variables inside cells. The interpolations are based on weighted least-square polynomial reconstructions inside partially biased stencils (collection of cells centered on the left, respectively right cell on either side of the interface).

The degree of the reconstructed polynomial is the highest enabled by the number of cells in the stencil (number of monomials *1.5), depending on the successive neighbours insertion (ref 1.).

Relevant solvers

Time accurate solutions either

- Non-linear implicit by dual time-stepping (3rd order in real time, Explicit RK for pseudo-time inner iterations), or
- Explicit Runge-Kutta options from 3 to 6 stages.

RK4 was chosen for all runs of this case 1.6.

High-order capability

K-exact reconstruction coded and verified up to a 4th degree full base of monomials, for quite regular patterns of unstructured triangles, tetrahedra, hexahedra.

High order for linear convection : Assertion of the spatial order of convergence for the transport by the upwind-biased convective operator of a sine-wave in a 3-periodic cube, cfl = 1.
4th degree full base polynomial reconstruction => Order of spatial convergence = 4.8
3rd degree full base polynomial reconstruction => Order of spatial convergence = 3.4
2nd degree full base polynomial reconstruction => Order of spatial convergence = 2.9

Parallel capability

- Loop-based Open-MP programming.

For the runs of the test-case 1.6, an acceleration between 9.2 and 11.2 was recorded, on a dual Westmere board (12 cores, 12 OMP threads) with respect to a single Westmere core.

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Post-processing

- Output in TecPlot ™ Format. Options for 2 types of visualisation :

  1/ Interpolation of the volume average primary variables to the nodes (in 2 stages : interpolation to face averages using the reconstruction/projection coefficients, then interpolation from face to node values). Visualisation by linear interpolation within the elements done by Tecplot.

  2/ Computation of the reconstructed polynomial fields to the nodes of triangular sub-elements within the cells, then visualisation by linear interpolation within the sub-elements done by Tecplot.
- Internal binary format for visualizations inside the GUI.

2. Case summary

Machines used (number of cores if parallel) : Dual Westmere Board, 12 cores, 12 OMP threads activated
Tau-bench wall clock times (sequential) on machines used : 7.84s.
The Work load (in Tau_bench Work units) are computed as the wall clock time multiplied by 12, divided by the Tau-Bench wall clock time (sequential).

3. Meshes

Description of meshes used for the case.
1/ Grids provided by the case coordinator (cartesian, randomly perturbed cartesian)
2/ New set of triangular grids (same number of cells as the cartesian ones, for each refinement level).

Case run with 4 grid refinements :
Very coarse : 32*32 quads (948 triangles),
Medium coarse : 64*64 quads (3962 triangles),
Medium fime : 128*128 quads (15980 triangles),
Very fine : 256*256 quads (64252 triangles).

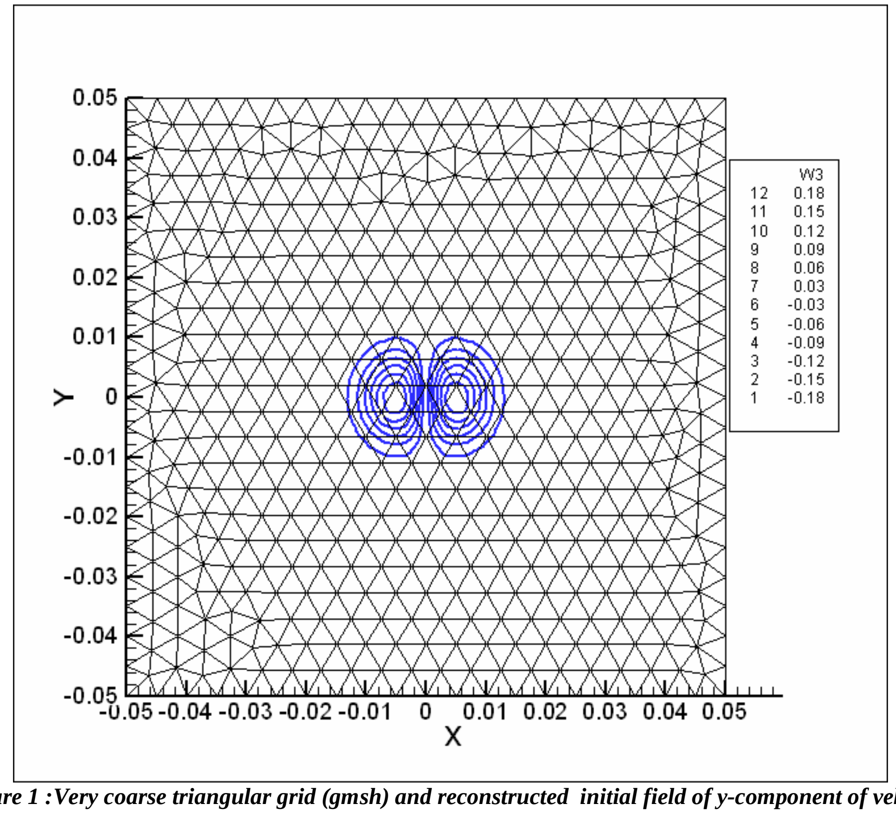
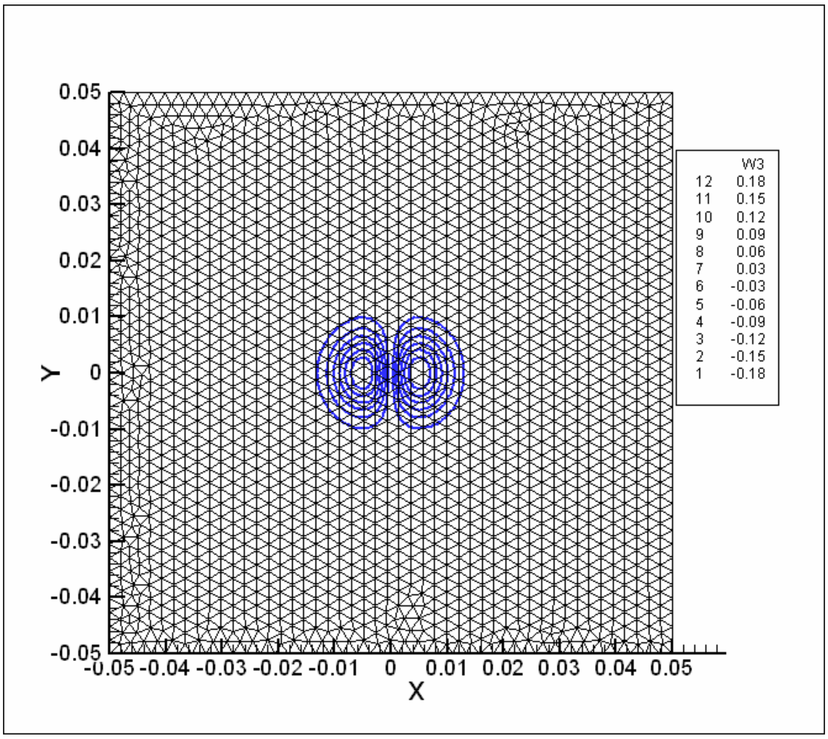


Figure 1 :Very coarse triangular grid (gmsh) and reconstructed initial field of y-component of velocity



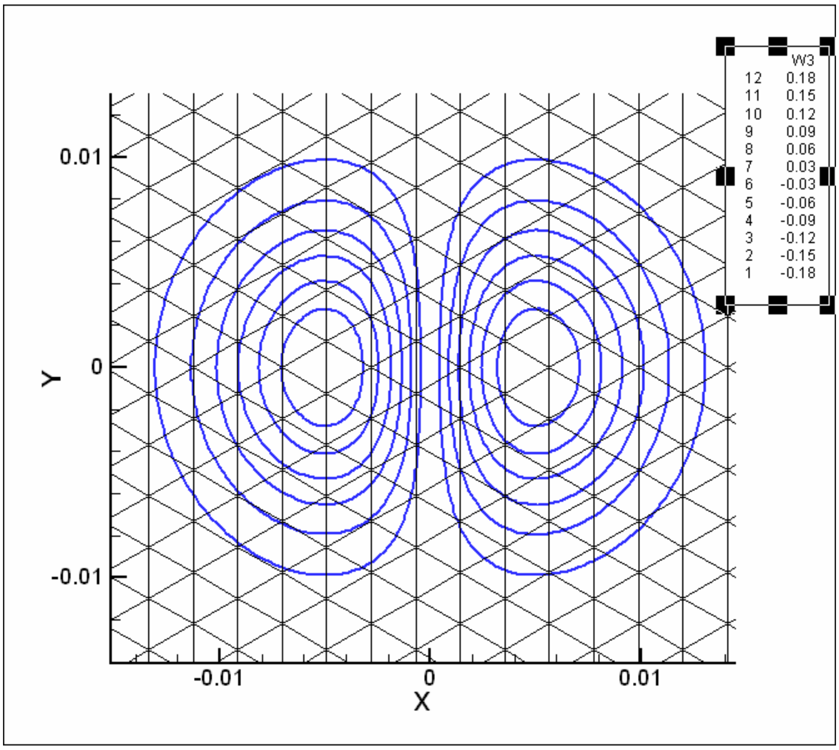


Figure 2 : Medium coarse triangular grid (gmsh, 3962 triangles) and reconstructed initial field of y-component of velocity Close-up

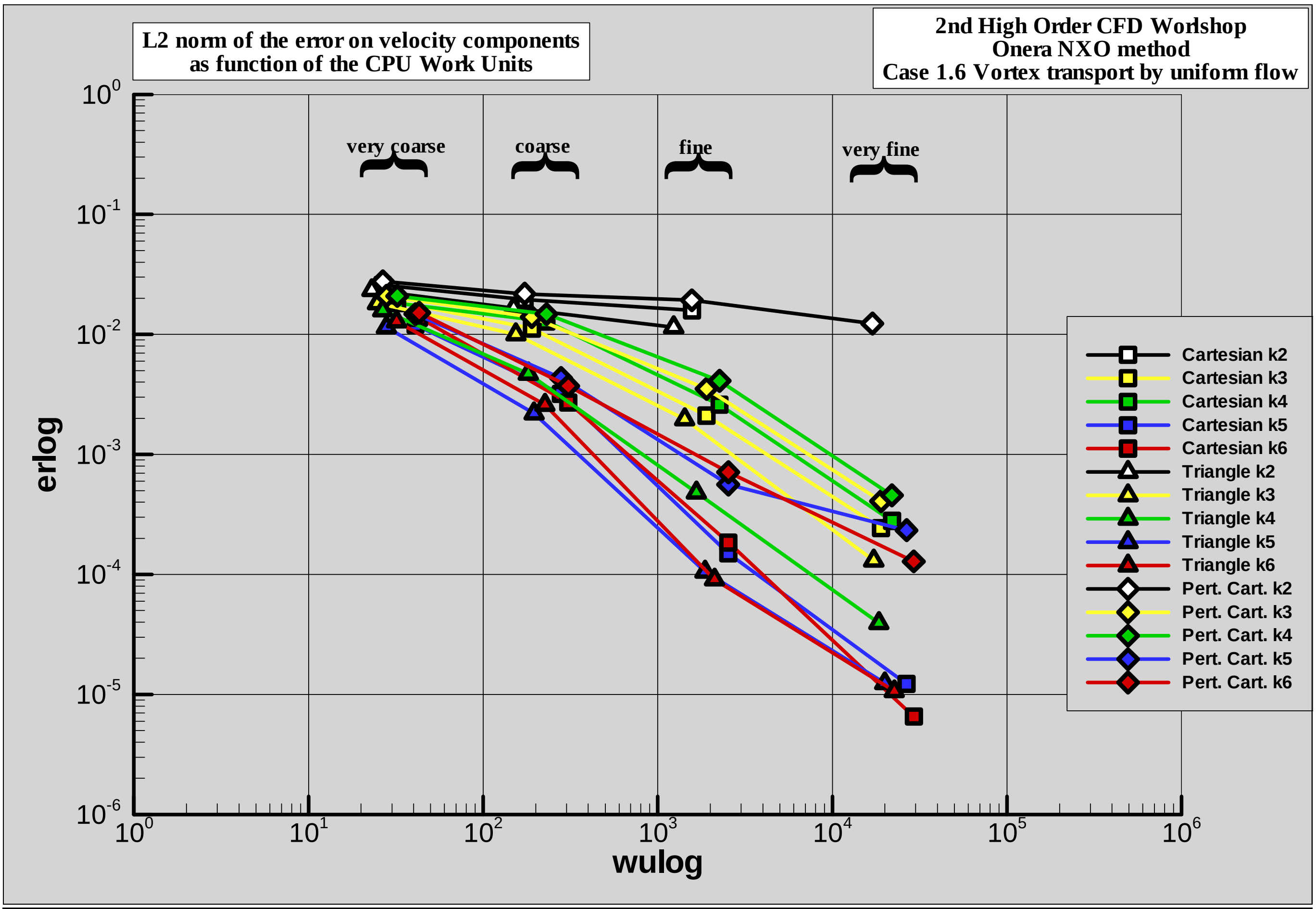
<u>Domain size</u>: as requested]-0.05,0.05[**2
<u>Physical simulation time</u> : 0.288 s (50 crossings)
<u>Time steps</u>

Triangles	grid 1 : 30000 dt, grid 2 : 60000 dt, grid3 : 120000 dt, grid4 : 240000 dt	==> cfl =1.12
Squares and quads	grid 1 : 36000 dt, grid 2 : 72000 dt, grid3 : 144000 dt, grid4 : 288000 dt	==> cfl = 0.93

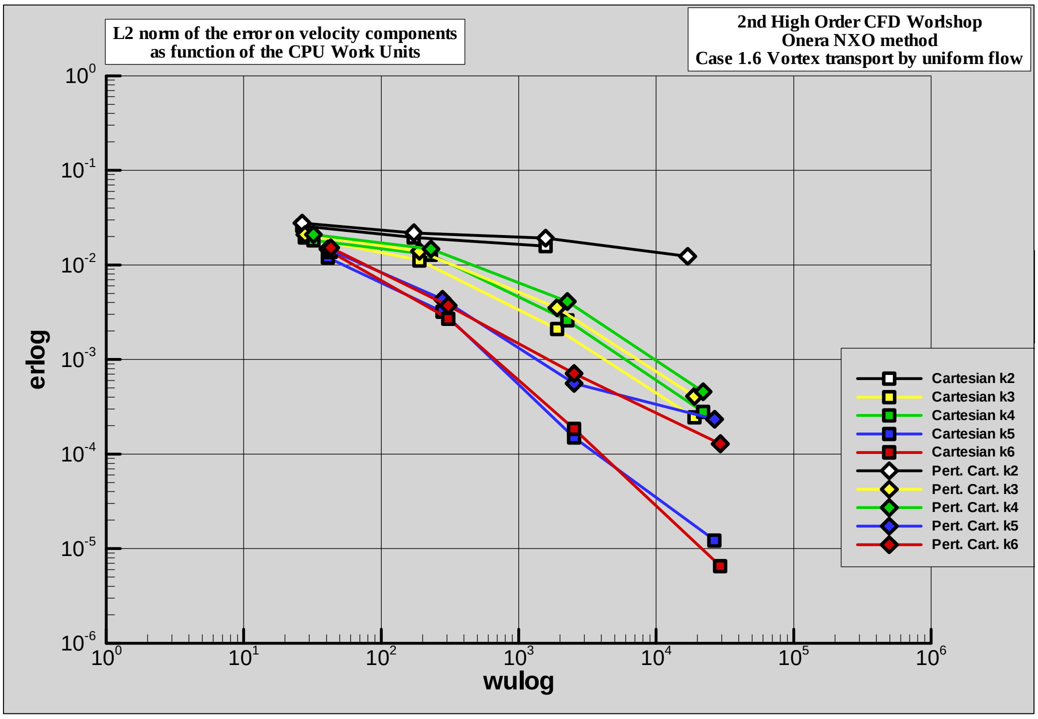
4. Results

Initialisation : The volume integral in each cell are computed by numerical integration with 9 significant digits. The initial reconstructed fields (high order polynomials in each cell with interface discontinuities) are verified in the visualisation package (figure 2).

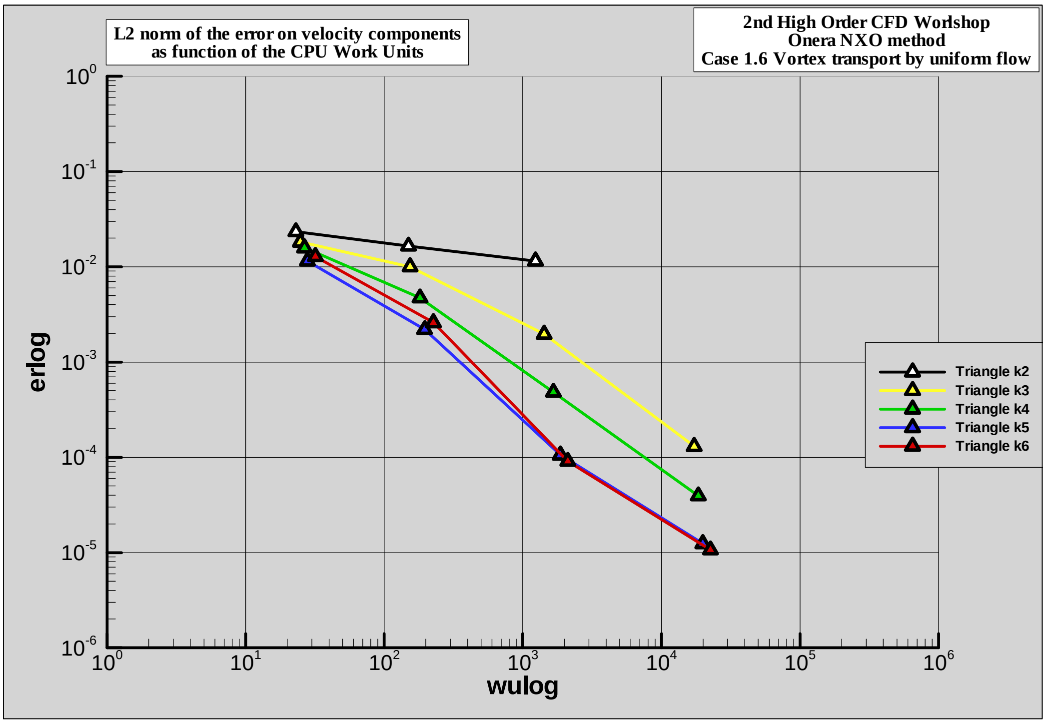
3 sets of results : cartesian, randomly perturbed cartesian,triangles (represented by square, diamond, triangle symbols)



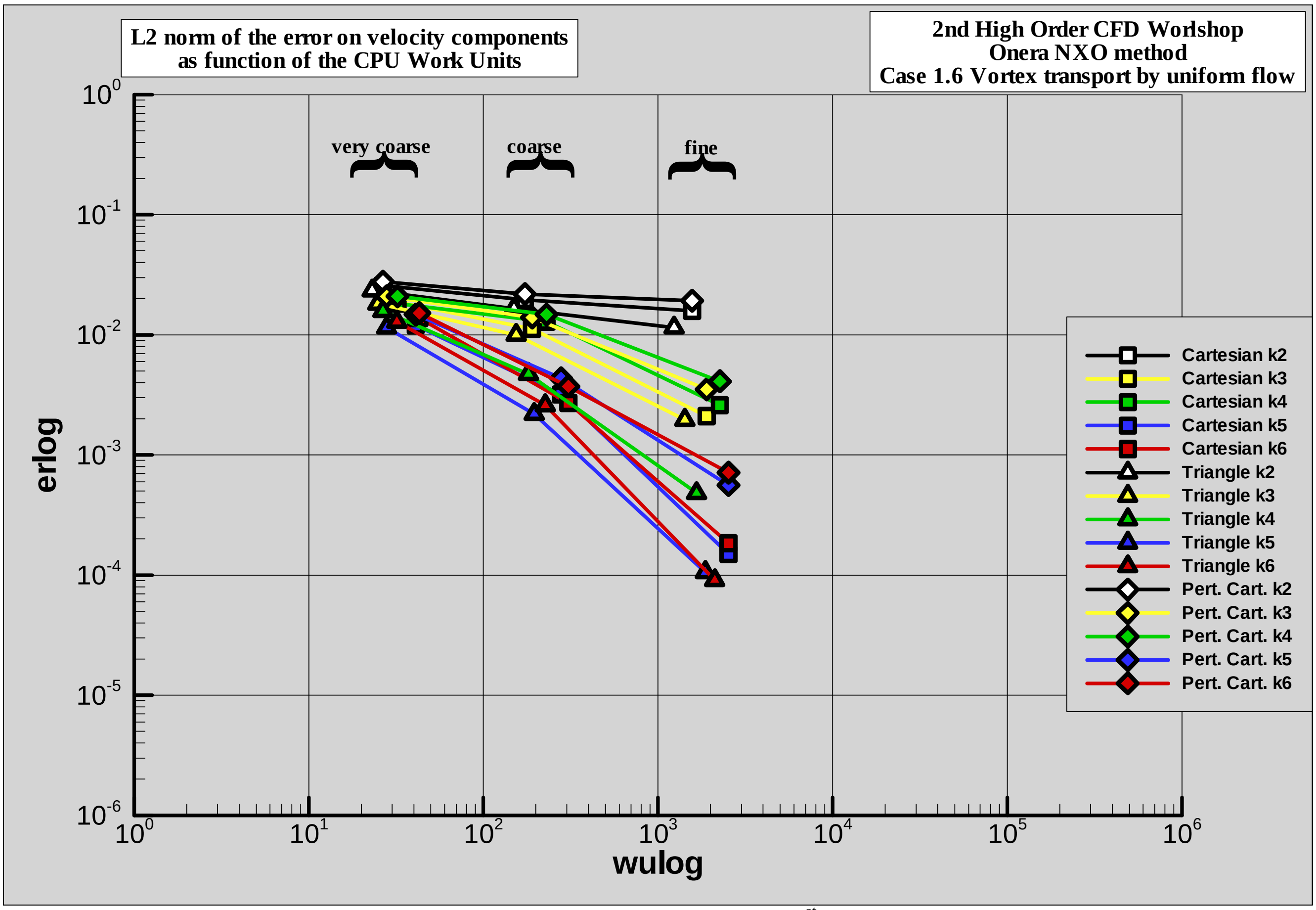
Full set of results



Results for cartesian and perturbed cartesian grids



Results for grids of triangles



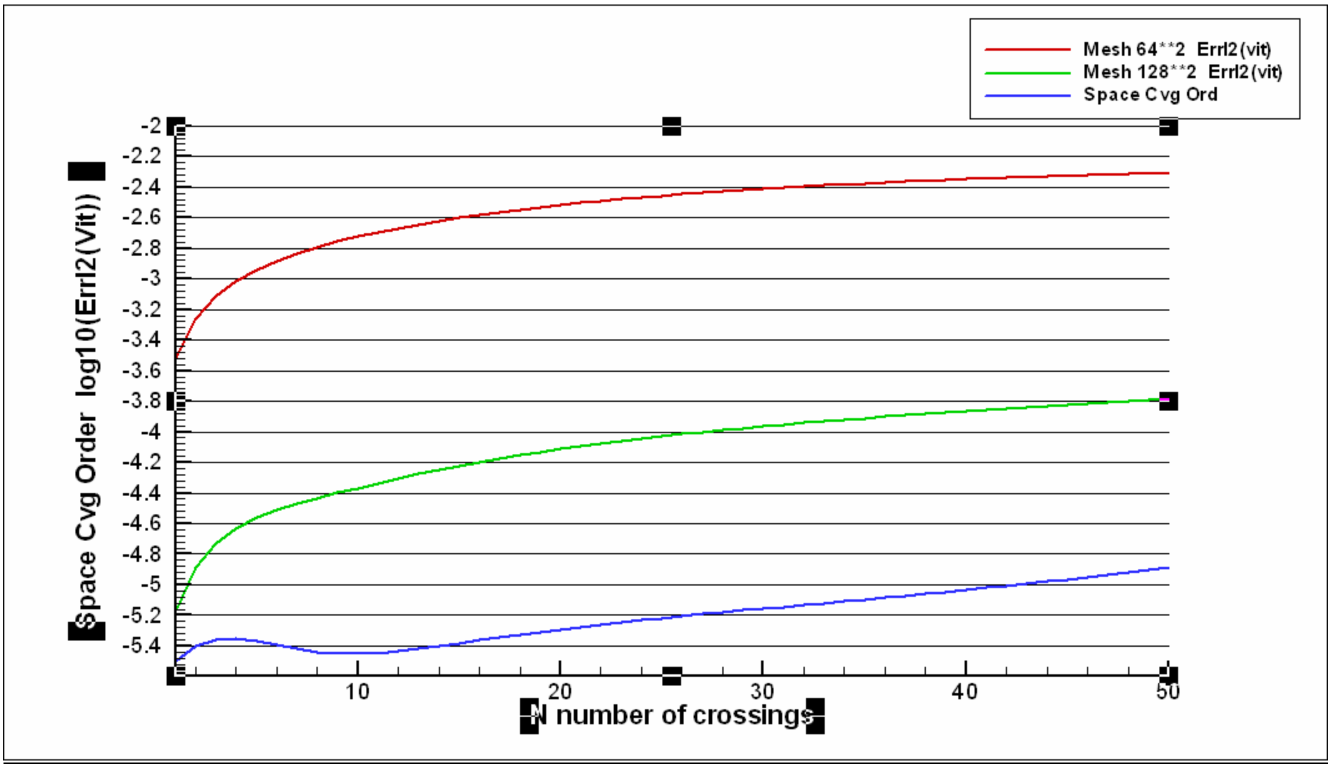
Results for the 3 coarser grids (as compared in the 1st HO CFD Workshop)

Tables of complete results. For each type of grid and scheme reconstruction degree, results with levels of refinement : Very Coarse, Coarse, Fine, Very Fine

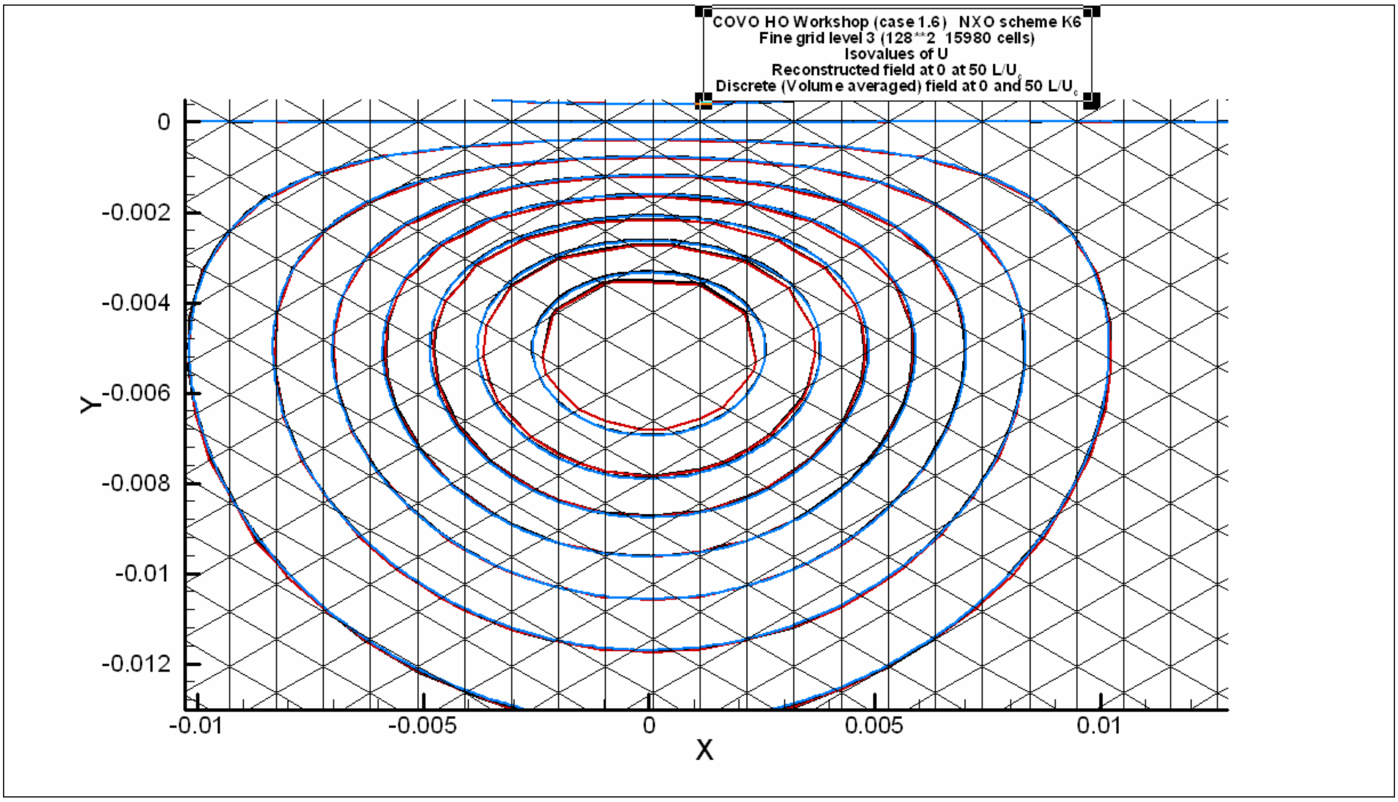
Cartesian grids	**Work Units**		**Error L2 / u,v**	**Conv. Order**
k2	**V.C.**	**26.5**	**2.57E-2**	**0.40** **0.30**
	C.	**173**	**1.95E-2**	
	F.	**1569**	**1.58E-2**	
k3	**V.C.**	**27.9**	**1.95E-2**	**0.80** **2.41** **3.12**
	C.	**190**	**1.12E-2**	
	F.	**1910**	**2.11E-3**	
	V.F.	**19250**	**2.43E-4**	
k4	**V.C.**	**32.2**	**1.82E-2**	**0.51** **2.31** **3.21**
	C.	**230**	**1.28E-2**	
	F.	**2260**	**2.59E-3**	
	V.F.	**21950**	**2.80E-4**	
k5	**V.C.**	**41**	**1.20E-2**	**1.91** **4.42** **3.63**
	C.	**279**	**3.20E-3**	
	F.	**2535**	**1.50E-4**	
	V.F.	**26595**	**1.22E-5**	
k6	**V.C.**	**43**	**1.38E-2**	**2.35** **3.88** **4.81**
	C.	**307**	**2.70E-3**	
	F.	**2535**	**1.84E-4**	
	V.F.	**29270**	**6.54E-6**	

Perturbed Cartesian grids	Work Units		Error L2 / u,v	Conv. Order
k2	**V.C.**	**26.5**	**2.76E-2**	**0.42**
	C.	**173**	**2.18E-2**	**0.18**
	F.	**1569**	**1.92E-2**	**0.64**
	V.F.	**16942**	**1.23E-2**	
k3	**V.C.**	**27.9**	**2.09E-2**	**0.59**
	C.	**190**	**1.39E-2**	**1.98**
	F.	**1910**	**3.52E-3**	**3.11**
	V.F.	**19250**	**4.08E-4**	
k4	**V.C.**	**32.2**	**2.09E-2**	**0.50**
	C.	**230**	**1.48E-2**	**1.85**
	F.	**2260**	**4.10E-3**	**3.17**
	V.F.	**21950**	**4.56E-4**	
k5	**V.C.**	**41**	**1.48E-2**	**1.77**
	C.	**279**	**4.34E-3**	**2.95**
	F.	**2535**	**5.61E-4**	**1.26**
	V.F.	**26595**	**2.34E-4**	
k6	**V.C.**	**43**	**1.52E-2**	**2.03**
	C.	**307**	**3.72E-3**	**2.39**
	V.F.	**2535**	**7.11E-4**	**2.47**
	V.F.	**29270**	**1.28E-4**	

Triangle grids	Work Units		Error L2 / u,v	Conv. Order
k2	**V.C.**	**23**	**2.34E-2**	**0.50**
	C.	**150**	**1.65E-2**	**0.52**
	V.F.	**1234**	**1.15E-2**	
k3	**V.C.**	**25**	**1.82E-2**	**0.94**
	C.	**154**	**1.00E-2**	**2.35**
	F.	**1425**	**1.96E-3**	**3.90**
	V.F.	**17270**	**1.31E-4**	
k4	**V.C.**	**27**	**1.58E-2**	**1.74**
	C.	**182**	**4.72E-3**	**3.29**
	F.	**1662**	**4.82E-4**	**3.61**
	V.F.	**18470**	**3.93E-5**	
k5	**V.C.**	**28**	**1.15E-2**	**1.98**
	C.	**196**	**2.19E-3**	**4.37**
	F.	**1865**	**1.06E-4**	**3.10**
	V.F.	**19943**	**1.24E-5**	
k6	**V.C.**	**32**	**1.28E-2**	**2.31**
	C.	**226**	**2.59E-3**	**4.83**
	F.	**2112**	**9.07E-5**	**3.10**
	V.F.	**22593**	**1.06E-5**	



Evolution of the space convergence order with time.
the time error is low with respect to space error but not completely cancelled.



Reconstruction k5, fine grid (final error L2 = 1.06E-4)
2 representations of the initial and final fields of x-component of velocity (superimposed)
1/ Volume averages interpolated to the nodes ==> linear interpolation in the elements : isolines represented by segments
2/ Reconstructed polynomial fields within the elements : smooth isolines

Comments :

Better efficiency (accuracy per work unit) for triangular grids

For squares and arbitrary quads, the reconstructions at 3rd and 4th degree have similar performances, the same holds for the 5th and 6th degree reconstructions (they use the same stencils).

For triangles, the accuracy increases with the degree of reconstruction gradually, but stalls at the 5th degree.

On a given grid :

- the number of dof per equation is equal to the number of cells, for all reconstruction degrees,
- the total memory occupation increases due to the extension of the stencil width : ratio of 1.7 from degree 2 to 6,
- the number of flop increases by a factor 1.4 from degree 2 to 6,
- the number of memory accesses increases by a factor 2.5 from degree 2 to 6,
- the Cfl number achievable does not vary with the degree of the reconstruction,
- The CPU work load (memory access + flop) increases by a factor 1.8 from degree 2 to 6

Reference 1 : **J.-M. Le Gouez, V. Couaillier, F. Renac**
High Order Interpolation Methods and Related URANS Schemes on Composite Grids.
48th AIAA Aerospace Sciences Meeting -Orlando, -USA (04-07 Jan 2010), AIAA-2010-513

2nd High Order CFD Workshop

Case 1.6 Vortex Transport by Uniform Flow
NXO method

Jean-Marie Le Gouez

Special thanks to Alain Lerat from ENSAM for his advice

CFD and Aeroacoustics Department
Fluid Mechanics and Energetics Branch

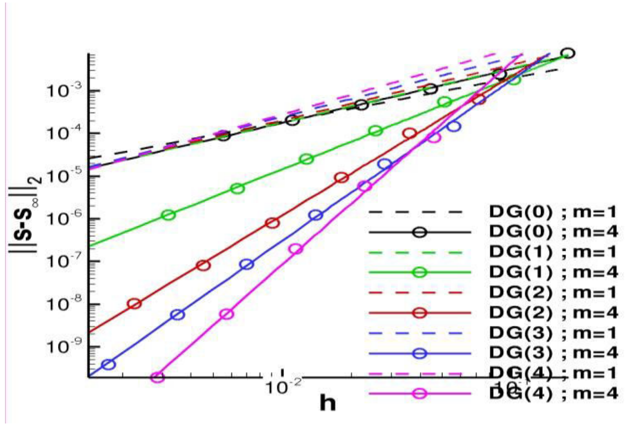
ONERA
THE FRENCH AEROSPACE LAB

retour sur innovation

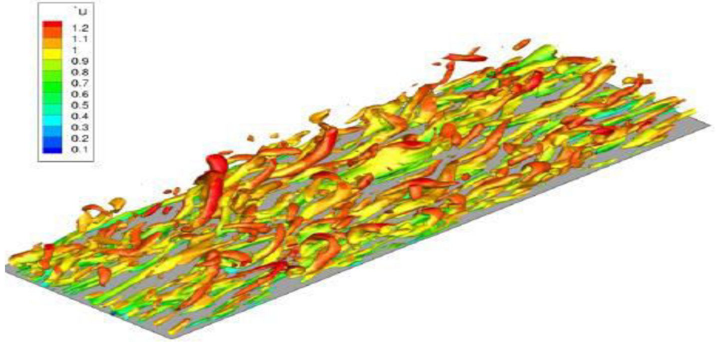
Reminder : **Aghora** Onera DG code prototype for high order simulations on complex flow physics
NXO : Prospective work on Reconstructed FV method on arbitrary grids

- AGHORA :
- Deployment of DG methods for high accurate simulations
- Modal / nodal approaches, High order meshes, hybrid polyhedral meshes,…
- Turbulence : RANS, LES (VMS_Subgrid), DNS
- New software architecture for dynamic adaptation and composite mesh on hybrid HPC architectures
- Adaptation : H (mesh), P (shape functions), M (modeling)

1st HO CFD Workshop : 3.5, Taylor-GREEN Vortex DNS and 1.1, smooth bump Euler



Test case C1.1 : Entropy error vs. mesh size
p = 0,...,4 ; linear mesh (m=1) and curved (m=4)
1rst HO workshop, Nashville, 2012



VMS-LES channel flow Rey=3000

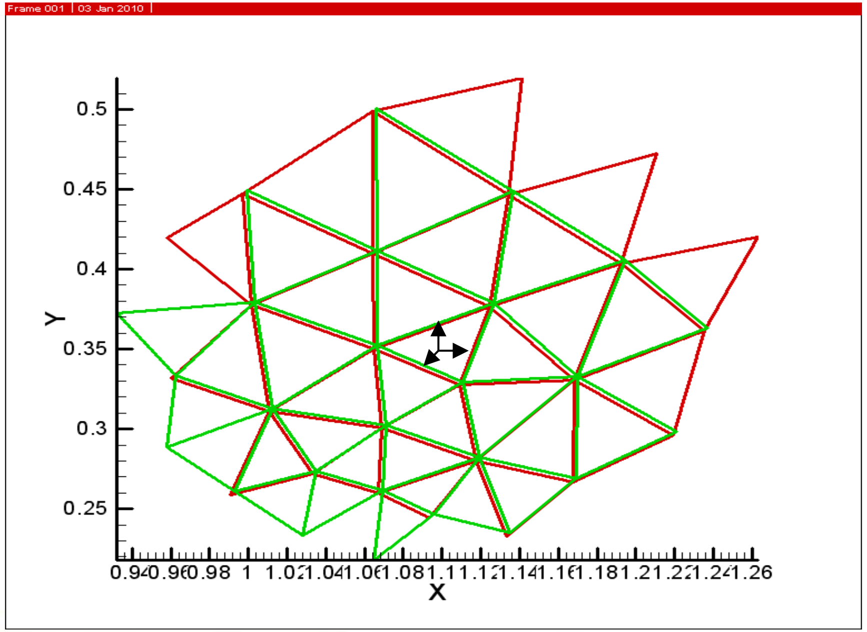
NXO : Prospective work on Reconsructed FV method on arbitrary grids

- 1 dof per cell and per equation
- Non compact scheme
- Parallel programming on node in shared memory using Open-MP (64 threads on SGI UV)
- MPI & coarse partitioning *foreseen* only for inter-node communication

- Reconstruction algorithm for the conservative variables fields and projection of the interfaces, single flux evaluation

All in the preprocessor : Weighted Least-Square polynomial fit adapts to the “quality” of the stencils

➔Gives the interpolation coefficients from volume averages to surface averages



Different stencils and polynomial degrees for the convective and the diffusive operators

Upwind-biased convective scheme based on characteristic splitting

Explicit RK time integration or Dual Time-stepping

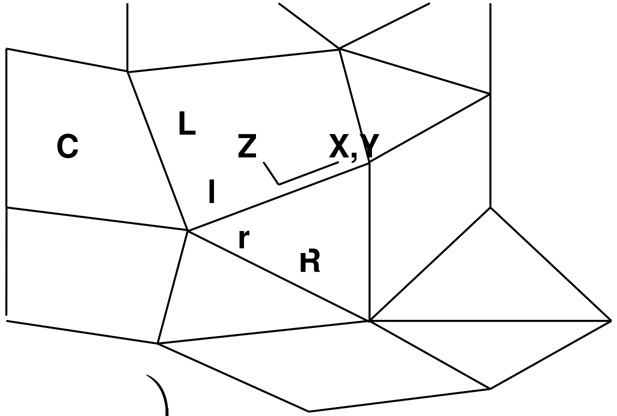
FV NXO method : Reconstruction and projection

$$\phi_a(X,Y,Z) = a_{\{ijk\}} X^i Y^j Z^k$$

$$\psi = \sum_{s=1}^{ns} \varpi_s \left(\Omega_{c(s)} \bar{\phi}_{c(s)} - \int_{\Omega_{c(s)}} \phi_a(X,Y,Z)\,dV \right)^2$$

Reconstruction error functional

Ns = Stencil size : nb of monomials + 50%



$$\hat{\phi}_l = \sum_{c=1}^{ns} \lambda_c \bar{\phi}_c$$

$$\hat{\nabla}\phi_l = \sum_{c=1}^{ns} \mu_c \bar{\phi}_c$$

$$\frac{\partial \psi}{\partial a_{\{ijk\}}} = \sum_{s=1}^{ns} \varpi_s \left(-2\Omega_{c(s)} \bar{\phi}_{c(s)} \Re^{ijk}_{c(s)} + 2a_{\{ijk\}} \Re^{ijk\,2}_{c(s)} + 2\Re^{ijk}_{c(s)} \sum_{\{ijk\}\neq\{i'j'k'\}} a_{\{i'j'k'\}} \Re^{i'j'k'}_{c(s)} \right) = 0$$

$$\Re^{ijk}_c = \int_{\Omega_c} X^i Y^j Z^k\,dV$$

Volume moment of order ijk

$$\hat{\phi}_L = \frac{1}{S_{LR}} \int_{\partial\Omega_{LR}} \phi_a(X,Y,Z)\,dS = \frac{\int_{\partial\Omega_{LR}} X^i Y^j Z^k\,dS}{S_{LR}} a_{\{ijk\}} = \nu_{\{ijk\}} a_{\{ijk\}}$$

$$\hat{\nabla}\phi_L = \frac{1}{S_{LR}} \int_{\partial\Omega_{LR}} \nabla\phi_a(X,Y,Z)\,dS = \left(i\,\nu_{\{i-1\,jk\}} e_X + j\,\nu_{\{ij-1k\}} e_Y + k\,\nu_{\{ijk-1\}} e_Z \right) a_{\{ijk\}} = \eta_{\{ijk\}} a_{\{ijk\}}$$

$$\hat{\phi}_L = \nu_{\{ijk\}} a_{\{ijk\}} = \nu_{\{ijk\}} \kappa_{g\,\{ijk\},c} \bar{\phi}_c = \lambda \bar{\phi}_c$$

$$\hat{\nabla}\bar{\phi}_L = \eta_{\{ijk\}} a_{\{ijk\}} = \eta_{\{ijk\}} \kappa_{g\,\{ijk\},c} \bar{\phi}_{c(s)} = \mu \bar{\phi}_c$$

Description of meshes used for the case.

1/ Grids provided by the case coordinator (cartesian, randomly perturbed cartesian)

2/ New set of triangular grids (same number of cells as the cartesian ones, for each refinement level).

Case run with 4 grid refinements :

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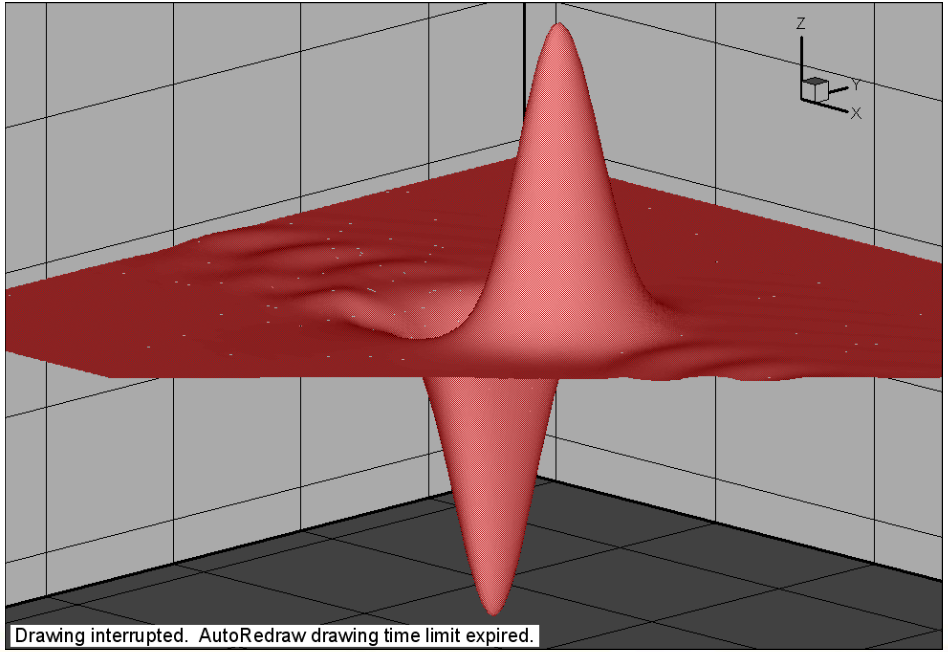
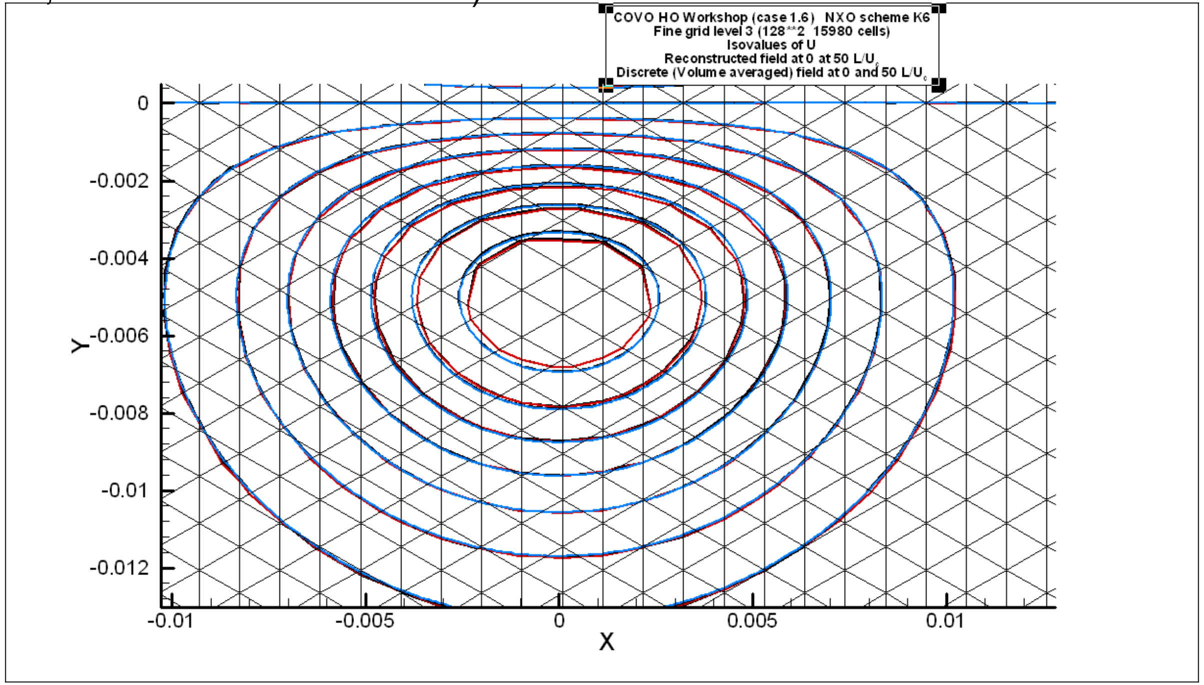
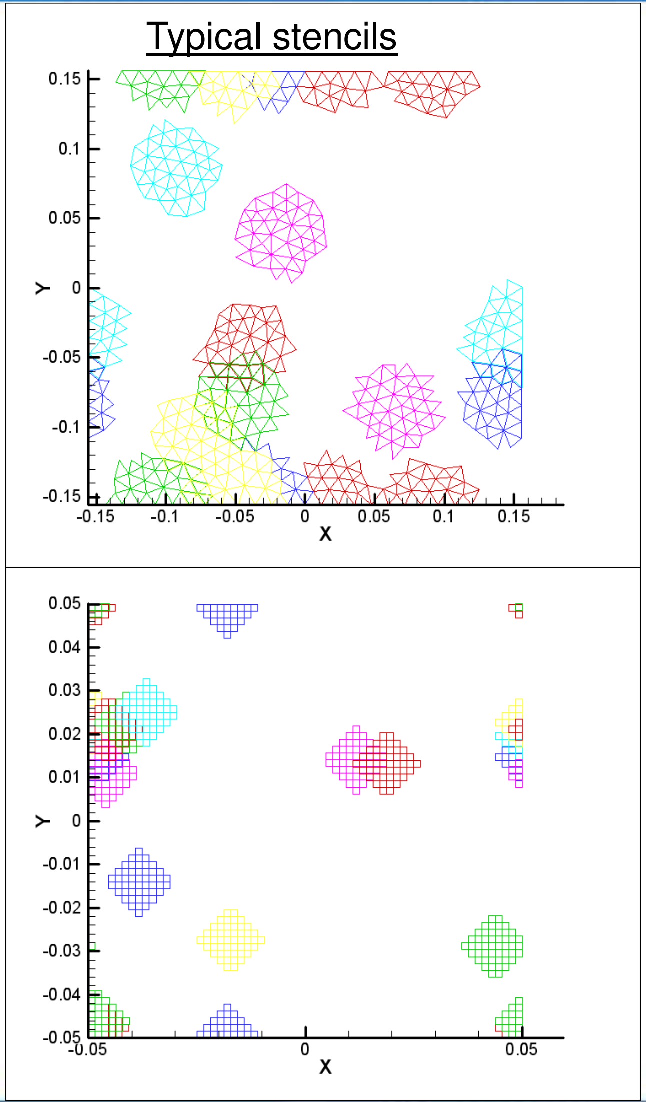


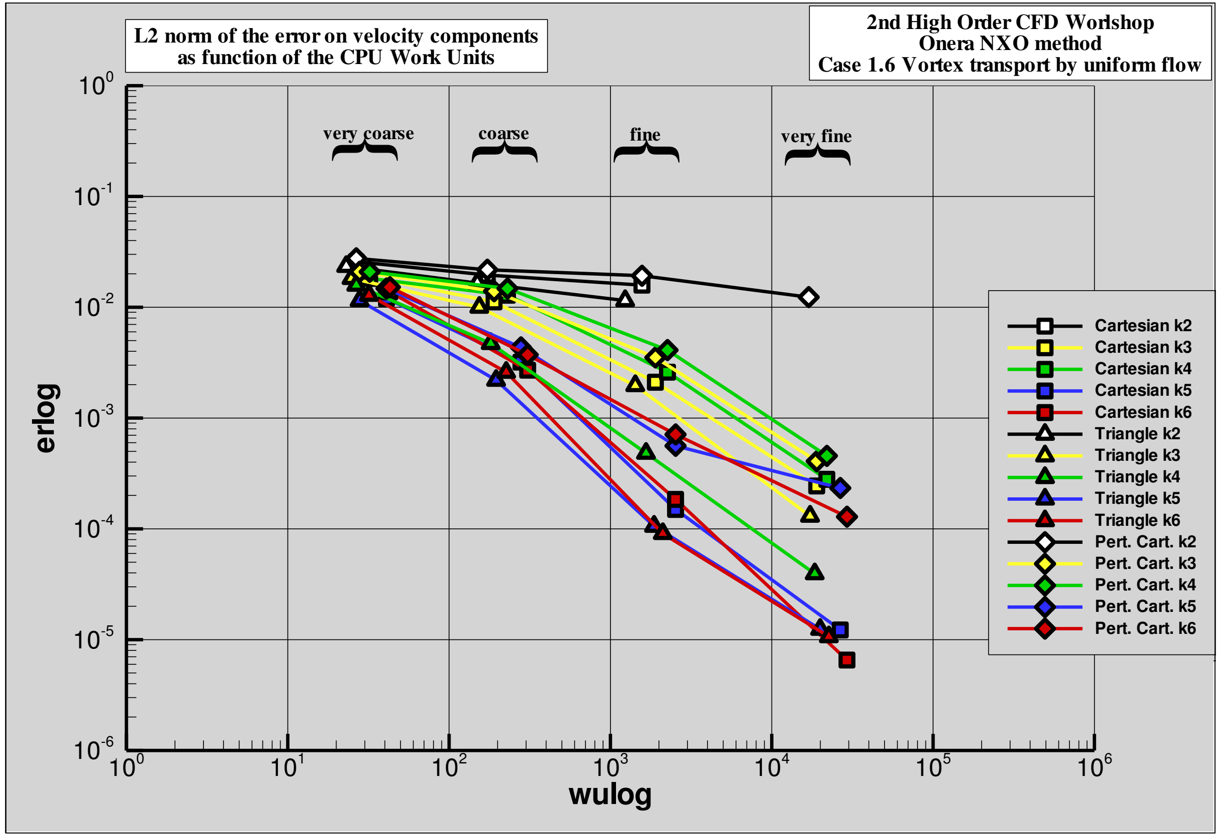


2 modes of visualization :

- Direct interpolation of the cell averages to the nodes
- Representation of the polynomial field in the central cell of each (convective) stencil

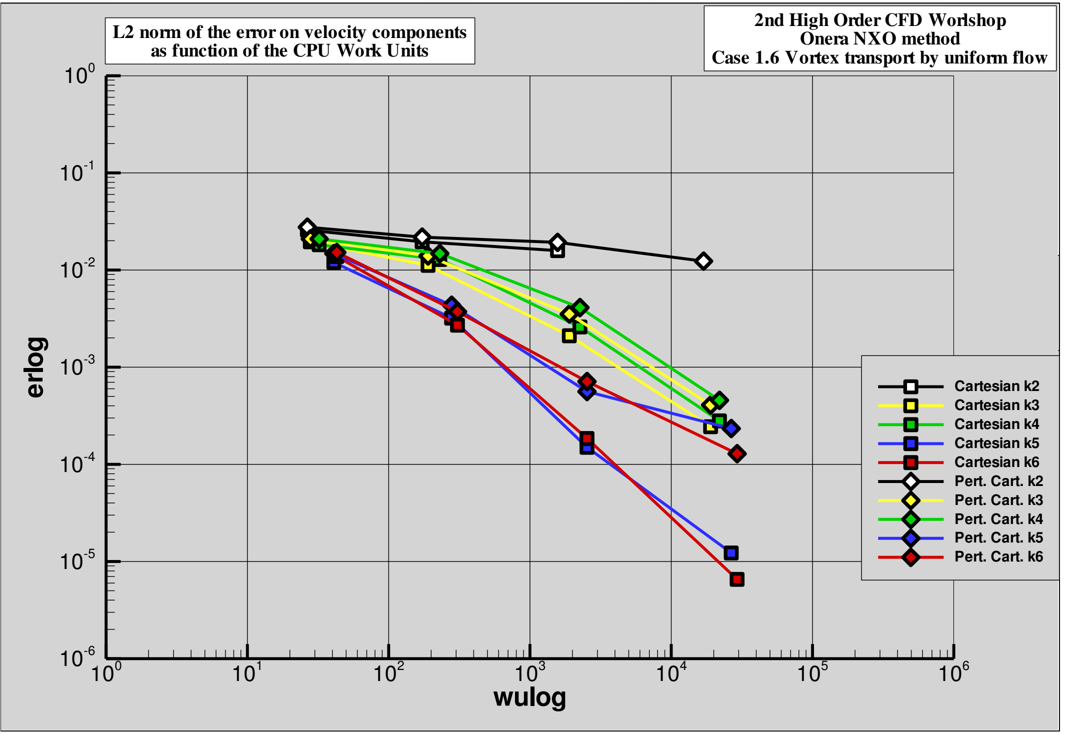


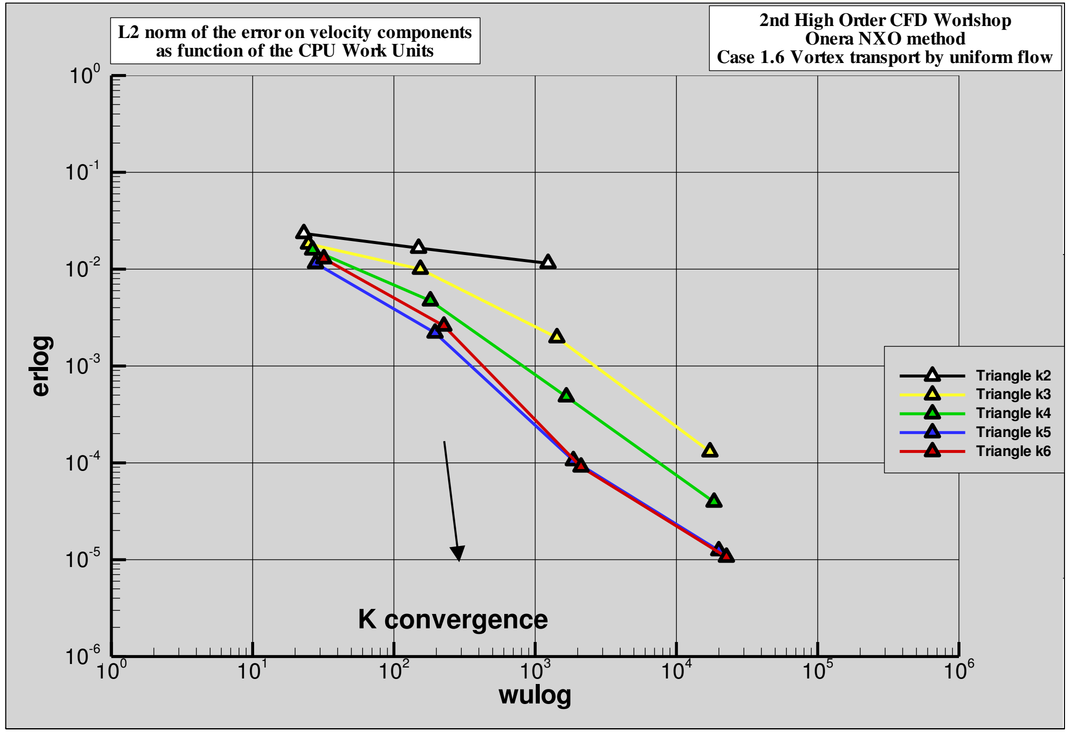
Typical stencils
Y
X



L2 norm of the error on velocity components as function of the CPU Work Units
2nd High Order CFD Worlshop
Onera NXO method
Case 1.6 Vortex transport by uniform flow
very coarse
coarse
fine
very fine
erlog
wulog
Cartesian k2
Cartesian k3
Cartesian k4
Cartesian k5
Cartesian k6
Triangle k2
Triangle k3
Triangle k4
Triangle k5
Triangle k6
Pert. Cart. k2
Pert. Cart. k3
Pert. Cart. k4
Pert. Cart. k5
Pert. Cart. k6

High Order CFD Workshop   Case 1.6 Vortex transport by uniform flow





Cartesian	CPU W.U.		Error	Cvg.
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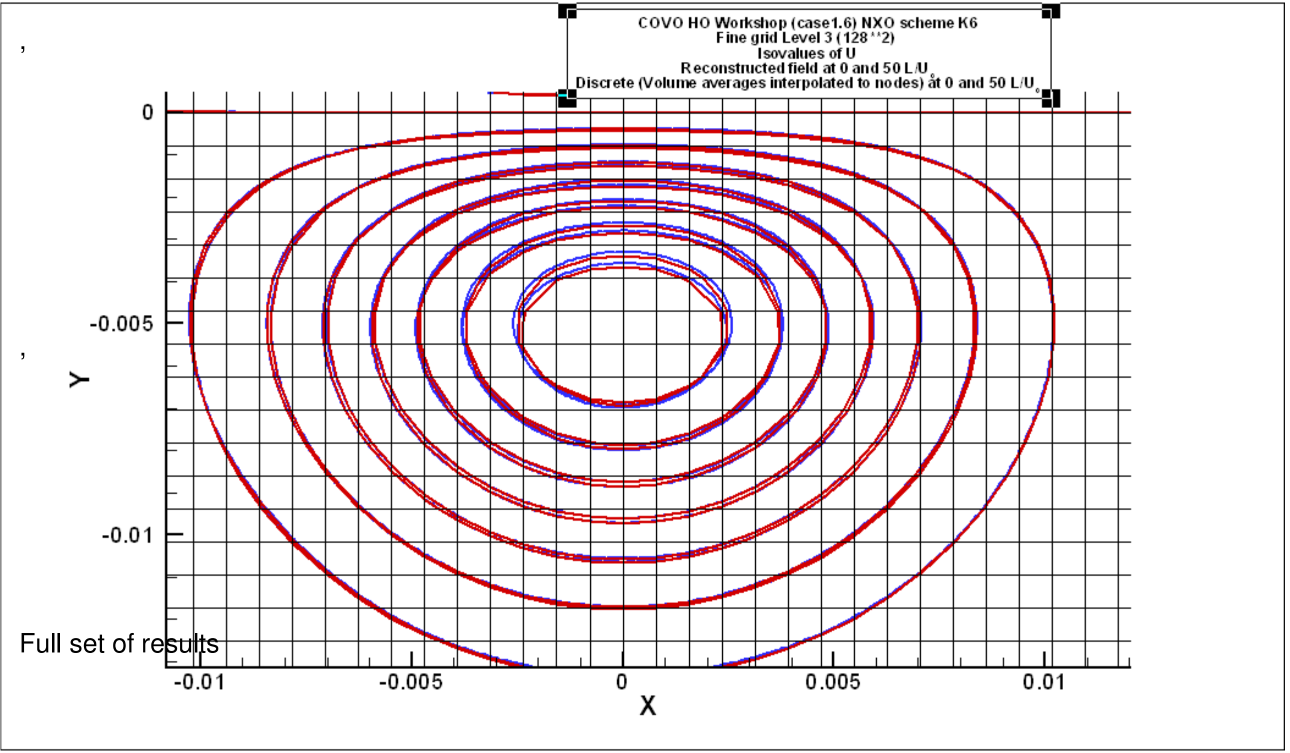
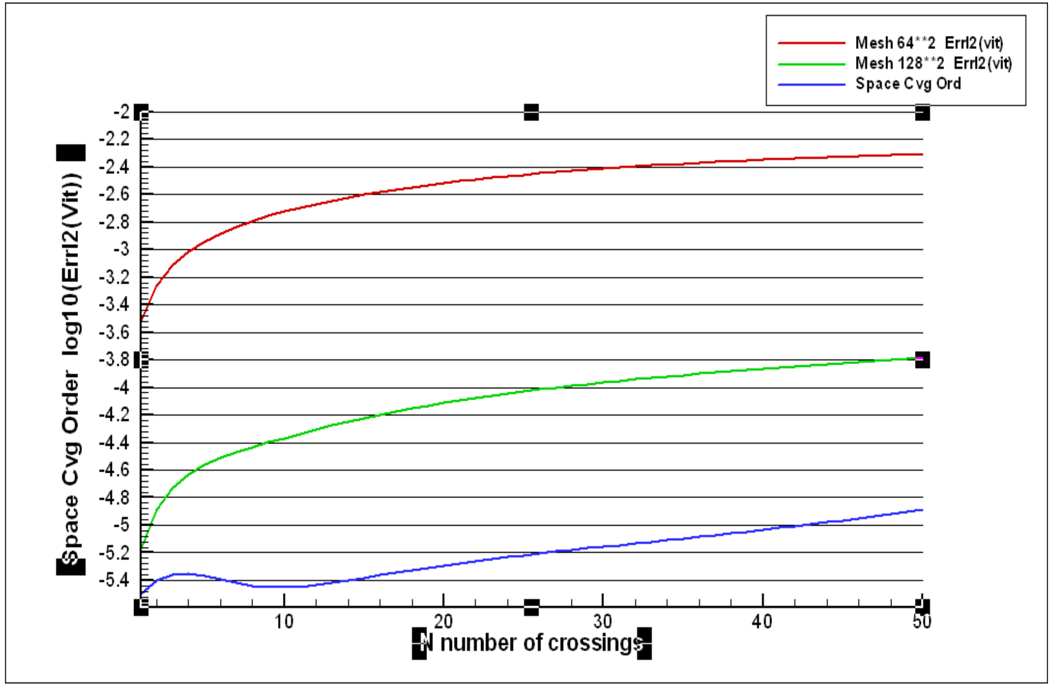
Initialisation : The volume integrals of CVs in each cell are computed by numerical integration with 9 significant digits



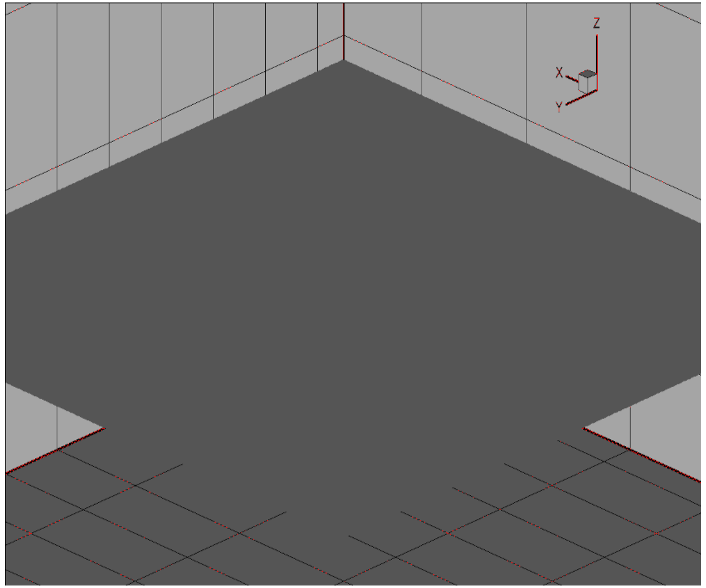
Full set of results



Space convergence order

Case 1.6 Vortex transport by uniform flow

Decomposition of the solution on the modes



Animation

Animation

Animation