Problem C2.2: Steady Turbulent Transonic flow over an Airfoil

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Outline

1. Govering equations / Flow conditions
2. Boundary conditions
3. Grids/reference solution
4. Results
   - Uniform grid refinement
   - hp-refinement
5. summary

Governing equations and Flow conditions

- 2D RANS equations with Wilcox kω-turbulence model
- transition fixed at 3%
- $\gamma = 1.4$, $Pr = 0.71$, $Pr_t = 0.9$
- $Ma_\infty = 0.734$, $Re = 6.5 \cdot 10^6$, $\alpha = 2.79^\circ$
- constant viscosity law $\mu = \sqrt{\gamma}\frac{Ma_\infty}{Re}$

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Boundary conditions

- farfield: vortex correction
- wall: adiabatic boundary condition with $\omega_W = \frac{6\mu}{\beta\rho(\alpha_p y_1)^2}$,
  whereas $\beta = 0.09$ and
  $\alpha_0 = 3.68 \cdot 10^{-1}$, $\alpha_1 = 8.21 \cdot 10^{-2}$,
  $\alpha_2 = 3.57 \cdot 10^{-2}$, $\alpha_3 = 1.99 \cdot 10^{-2}$
  Note: Menter proposed $\alpha = \frac{1}{\sqrt{10}} \approx 3.16 \cdot 10^{-1}$ for Finite Volume methods

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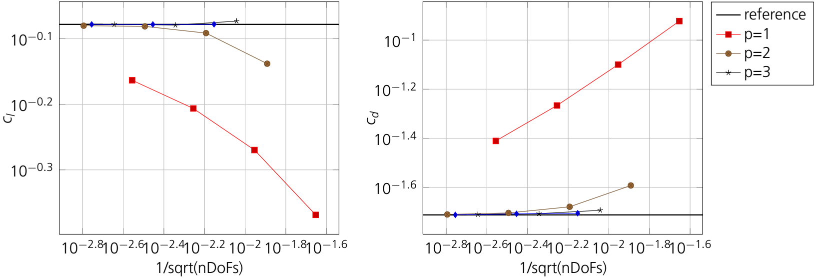
Grids and reference solution

- grids from website are used

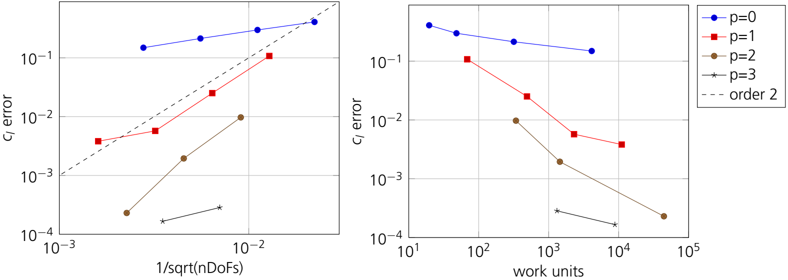
level	# cells	used polynomial degree
4	2024	0-3
3	8096	0-3
2	32384	0-3
1	128536	0-1

- 4^{th}-order solution on level 2 grid is used as reference solution:
  - $c_l^{(ref)} = 0.8353096597$
  - $c_d^{(ref)} = 0.0194058274$

convergence study



error study - c_l



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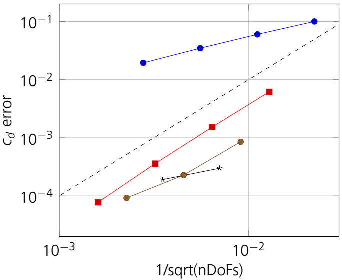
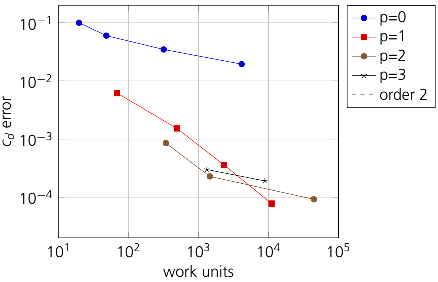
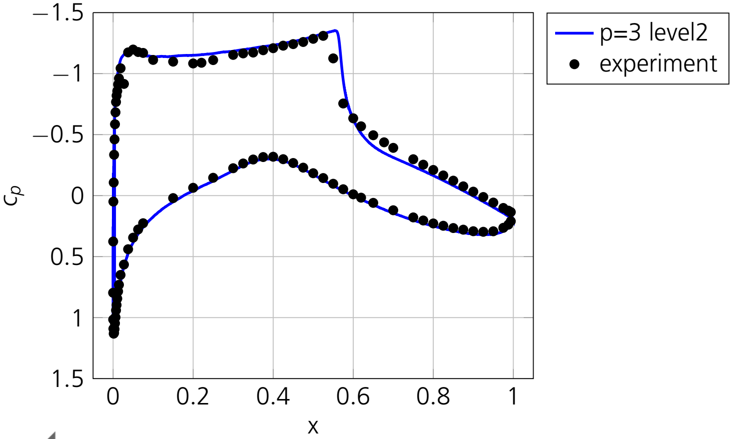
error study - c_d

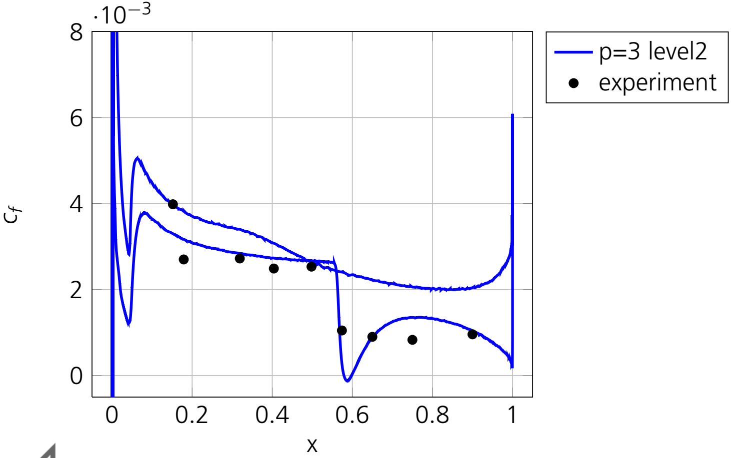




reference solution: c_p



reference solution: c_f

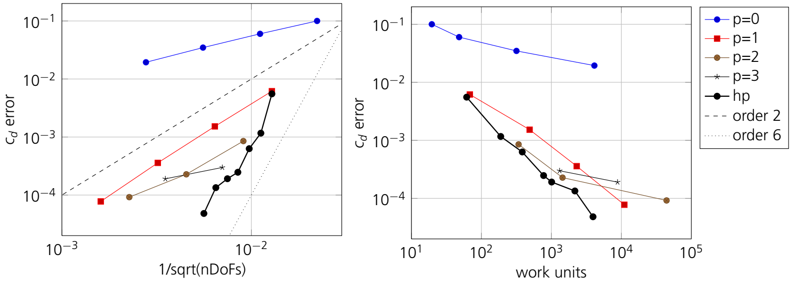


hp-adaptive results

Solution-adaptive mesh and order adaptation procedure

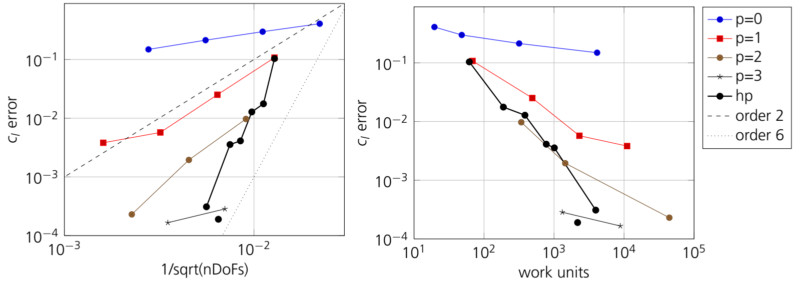
1. local error estimation
   - adjoint problem for a combination of c_l, c_d and c_m
   - linear solve with increased polynomial degree
   - fraction of elements with largest error estimate marked for refinement
2. selection between *p*-enrichment and *h*-subdivision
   - smoothness estimation based on spectral analysis: decay of Legendre series coefficients
3. anisotropic *h*-subdivision
   - based on directional decomposition of local error estimate

error study - c_d



polynomial degree	1	2	3	4	5	6	7
fraction of elements [%]	25.7	30.3	17.9	9.4	10.4	4.6	1.7
fraction of DoFs [%]	7.8	18.4	18.1	14.3	22.1	13.1	6.1

error study - c_l



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Summary

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- with uniform mesh refinement only 2^{nd} order can be achieved, because of the shock (capturing)
- hp-adaptive refinement strategy works like 6^{th} order, although more than the half of all elements are treated with polynomials of order 1 and 2

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Thank you for your attention

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