C3.5 Simulation of the Taylor-Green Vortex

2^{nd} International Workshop on High-Order CFD Methods

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Diablo Flow solver

- Solves the compressible three-dimensional **Euler/Navier-Stokes/RANS** equations on structured multiblock grids with C^0 continuity at block interfaces
  - Spalart-Allmaras 1-equation turbulence model (Currently second-order)
- Spatial discretization is obtained with high-order **Summation-by-Parts (SBP)** finite-difference operators
  - Second derivatives in the viscous fluxes can be computed with either the application of the first derivative twice or a compact-width-stencil operator
- Block interface coupling and boundary conditions are enforced with **Simultaneous-Approximation-Terms (SATs)**
- The nonlinear system is solved using an **Inexact Newton-Krylov** algorithm with a **Pseudo-Transient Continuation** startup phase
- The linear system is solved with **FGMRES** and a **Parallel Approximate-Schur Preconditioner**

Hicken, J. and Zingg, D.W., "A Parallel Newton-Krylov Solver for the Euler Equations Discretized Using Simultaneous Approximation Terms", *AIAA J.*, Vol. 46, No. 11, 2008, pp. 1695-1704

Osusky, M., Hicken, J. and Zingg, D.W., "A parallel Newton-Krylov-Schur flow solver for the Navier-Stokes equations using the SBP-SAT approach", *48th AIAA Aerospace Sciences Meeting*, 2010-116, Jan, 2010, Orlando, FL.

Del Rey Fernández, D.C. and Zingg, D.W., "High-Order Compact-Stencil Summation-By-Parts Operators for the Second Derivative with Variable Coefficients", *ICCFD7*, ICCFD7-2803, 2012

Diablo Flow solver

- General implementation for implicit and explicit **Multistep Runge-Kutta (MRK)** methods which specifically includes:
  - Linear multistep methods (Euler, BDF, Trapezoidal, . . .)
  - Runge-Kutta methods (Explicit RK, SDIRK, ESDIRK, . . .)
- Newton's method is accelerated for implicit time-integration methods using:
  - **Lagrange polynomial extrapolation** from step/stage solution values
  - **Delayed preconditioner updates** for individual stages or steps
  - **Relative tolerance termination of the nonlinear subiterations**
- The quadrature used to obtain integrated quantities is the high-order SBP norm consistent with the discretization
  - Superconergence of functionals if the discretization is dual-consistent and the solution is sufficiently smooth

Osusky, M., Boom, P.D., Del Rey Fernández, D.C. and Zingg, D.W., "An Efficient Newton-Krylov-Schur Parallel Solution Algorithm for the Steady and Unsteady Navier-Stokes Equations", *ICCFD7*, ICCFD7-1801, 2012

Hicken, J. and Zingg, D.W., "Summation-by-Parts Operators and High-Order Quadrature", *J. Comp. and App. Math.*, 237(2013), pp.111-125

Hicken, J. and Zingg, D.W., "Superconvergent Functional Estimates from Summation-by-Parts Finite-Difference Discretizations", *SIAM J. on Sci. Comp.*, Vol. 33, No. 2, 2011, pp. 893-922

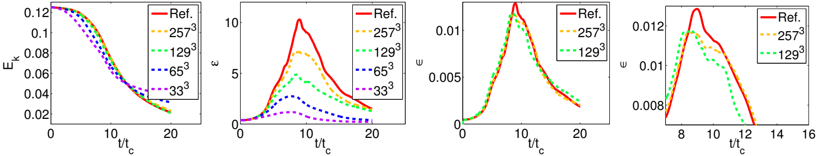
Simulation parameters

- Ratio of specific heats: $\gamma = 1.4$
- Prandtl number : $Pr = 0.71$
- Mach number: $M = 0.1$
- Reynolds number : $Re = 1600$
- Time Integration: ESDIRK2/TRBDF2
- Grid: Cartesian multiblock
- Block sizes: 33^3
- Results were generated with a one-to-one block to processor ratio
- TauBench reference time is 9.5968 sec

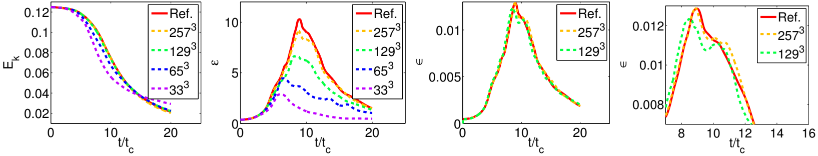
Computations were performed on the General Purpose Cluster (GPC) supercomputer at the SciNet HPC Consortium. SciNet is funded by: the Canada Foundation for Innovation under the auspices of Compute Canada; the Government of Ontario; Ontario Research Fund - Research Excellence; and the University of Toronto.

Full Solution Domain

Order (2,1,2) - Constant CFL of ∼ 15.8



Order (6,3,4) - Constant CFL of ∼ 25.3



Results

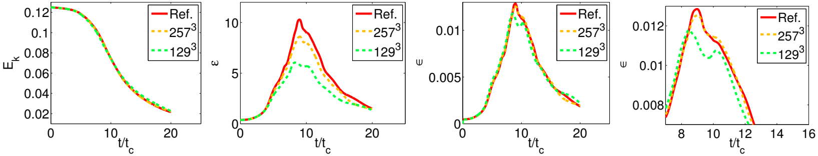
Full Workshop Domain

Grid size	$\overline{\text{Newton}}_{\text{its}}$/ time stage	$\overline{\text{GMRES}}_{\text{its}}$/ time stage	Walltime (sec.)	Work Units
Order (2,1,2) spatial discretization/Order (2,1,2) dissipation model				
33^3	5.0	12.9	2897.5	302
65^3	5.0	12.0	3200.6	2668
129^3	5.0	11.9	3251.0	21681
257^3	4.3	10.0	2939.6	156830
Order (6,3,4) spatial discretization/Order (7,3,4) dissipation model				
33^3	5.8	43.8	12318	1284
65^3	5.3	37.6	12881	10737
129^3	5.0	34.3	12048	80346
257^3	5.0	33.9	12365	659690

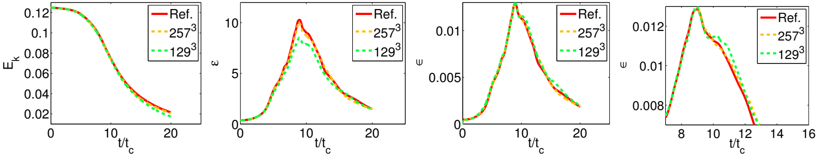
- An order (6,3,4) simulation with 129^3 nodes obtains similar results to an order (2,1,2) simulation with 257^3 nodes, but requires $\sim$ 49% less work units
- This reduction is increased to $\sim$ 69% when the order (5,2,3) dissipation model is used

One Octant Domain with Symmetry Boundary Conditions

Order (2,1,2) - Constant CFL of ~ 31.6



Order (6,3,4) - Constant CFL of ~ 50.5



Summary

- Require at least 129^3 nodes to reasonably approximate the dissipation of kinetic energy
- Even order (6,3,4) with 257^3 nodes under resolved the flow
- Comparable solutions are obtained using order (6,3,4) with 129^3 nodes, and order (2,1,2) with 257^3 nodes, but with 49% less computational expense
- Applying symmetry boundary conditions in one octant of the domain can artificially double your grid resolution

Questions?

Other cases submitted by our group:

C1.1 Internal inviscid flow over a smooth bump

Del Rey Fernández, Boom, Zingg, Hicken

C1.2 Transonic Ringleb flow

Del Rey Fernández, Boom, Zingg, Hicken

C1.5 Radial expansion wave

Boom, Del Rey Fernández, Zingg, Hicken

C3.3 Transitional flow over a SD7003 wing

Boom, Del Rey Fernández, Zingg, Hicken

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