DNS of Taylor-Green Vortex and Periodic Hill Flow by a High-Order Finite Difference Scheme

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1 Code description

Calculations in this work were carried out with a 4^{th}-order variant of the DLR FLOWer code. The basic FLOWer code solves the compressible Reynolds-averaged Navier Stokes equations on block-structured grids with second-order finite volume techniques. The high-order version, chosen for the present work [1], uses 4^{th}-order central differencing based on implicit compact finite differences in a cell-centered formulation, together with high-order compact filters, applied at the end of each time step. For the present investigation we use an 8^{th}-order filter at very low filter-constants to minimize influences on the DNS. Time advancement is obtained by a three-step second-order Runge Kutta method [2].

2 Case Summary

The Taylor-Green vortex and 2D Periodic Hill flow are investigated for testing the DNS capabilities of the code.

The Taylor-Green vortex at Re=1600 and M=0.1 is initialized by the standard analytic function from the workshop-description. Studies of different filter parameters and grid densities are carried out on up to 24 Cpu's in about 10 hours on the fine grid. No residual indicators are necessary for this test-case but the dissipation is extracted over simulation time and compared with the reference data.

The Periodic Hill testcase has been taken up as a standard benchmark in ERCOFTAC, and hence a thorough description can be found on the ERCOFTAC web-page. For the compressible FLOWer code a Mach number of 0.1 is chosen at a Reynolds Number of 2800 and a fixed dimensionless wall temperature of 1. Statistical convergence is guaranteed by the use of two different averaging periods. The simulation time before starting the averaging process was about 100 hours on 128 Intel Xeon Cpu's of the DLR-$C^2A^2S^2E$ cluster (fine grid) and the same calculation time is expected for generation of turbulent statistics.

3 Meshes

Grids for both test cases were provided by the respective workshop web-pages. The boundary conditions of the Taylor-Green vortex are periodicity in all directions. 128 equidistant structured cells were distributed over a cube of edge-size π. Due to the cubic equally-sized nature of the task, preservation of geometric fidelity is evident.

This is more difficult for the periodic hill, where curve-linear grids of 256x128x128 and 128x64x64 cells were used. These grids were provided for all Workshop-participants at sufficient quality of point-distribution and cell-orthogonality. For curve-linear meshes general transformations were used by the finite-difference code to provide calculations in a cartesian computational space.

4 Results

For both test cases, results are found well in the range of the provided references. An overview of the Taylor-Green Vortex at startup and after 20 convective time steps t_c is shown in Fig. 1 by vorticity iso-surfaces, colored by the z-vorticity . The resolution of the calculated turbulent structures is very similar to the illustration from the case description. Calculations of the Turbulent Kinetic Energy (TKE), the energy dissipation and the enstrophy show good agreement with the reference data. An overview of the time-averaged periodic-hill flow, including stream-traces and TKE is provided in Fig. 2. In comparison with literature-data, the TKE distribution is in the expected range between results at Re=700 and 10595. Comparisons with turbulent quantities from different cuts, as provided on the ERCOFTAC web page, are shown in the final presentation.

References

[1] S. Enk, "A Fourth Order Finite Difference Method for Large Eddy Simulation at a Flat Plate", 16. DGLR-Fach-Symposium der STAB, November 3-4, 2008, Aachen, Germany.

[2] Kroll, N., Jain, R.K., "Solution of Two-Dimensional Euler Equations- Experience with a Finite Volume Code," DFVLR Forschungsbericht 87-41, (1987).

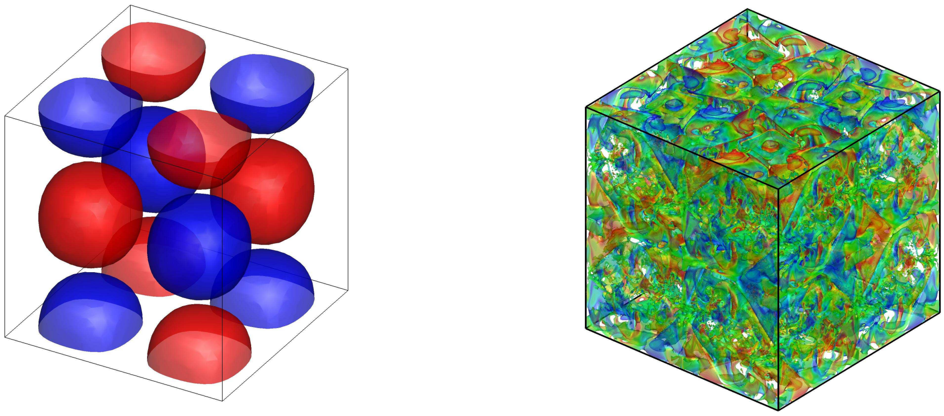

Figure 1: Illustration of Taylor-Green vortex at t = 0 (left) and t = $20\,t_c$ (right). Vorticity Iso-surfaces colored by the z-component of the dimensionless vorticity.

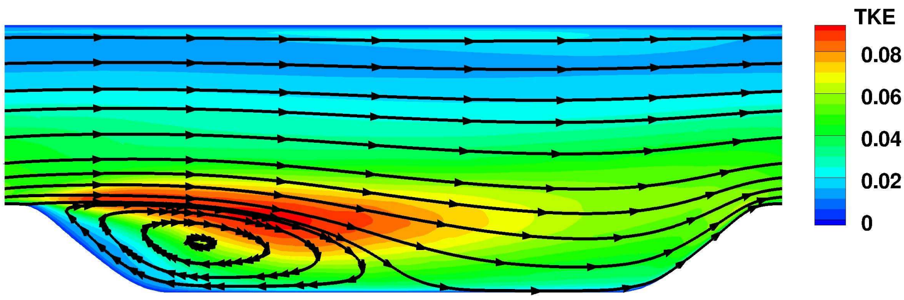

Figure 2: Streamtraces and turbulent kinetic energy computed on the time- and spanwise averaged velocity field. Simulation by 4^{th} order Pade-scheme and 8^{th} order implicit filter.