Eurpean Conference on Computational Fluid Dynamics
ECCOMAS CFD 2006

Time Dependent Adaptation for compressible flows

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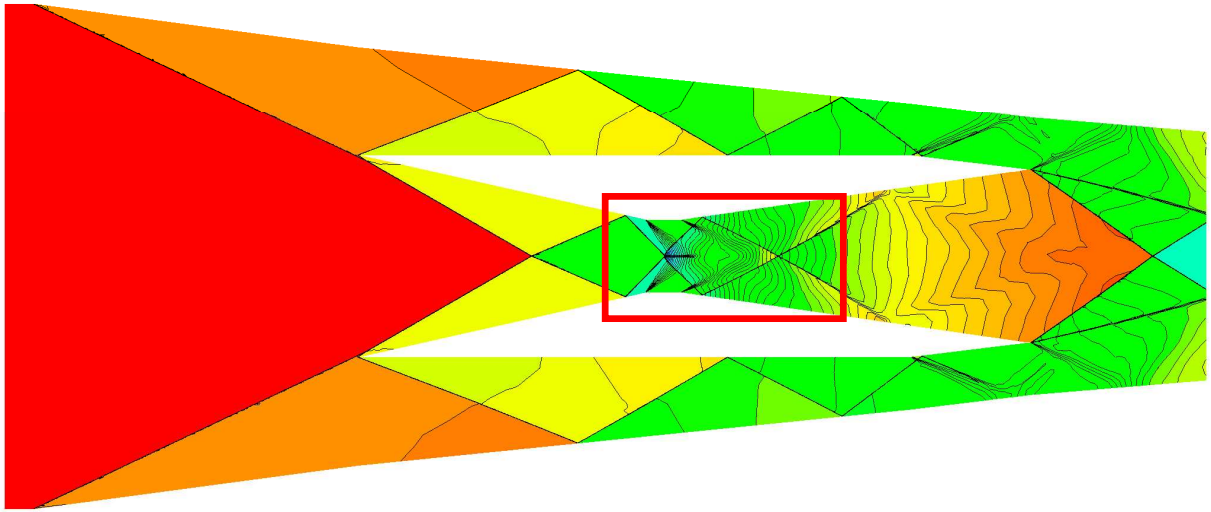
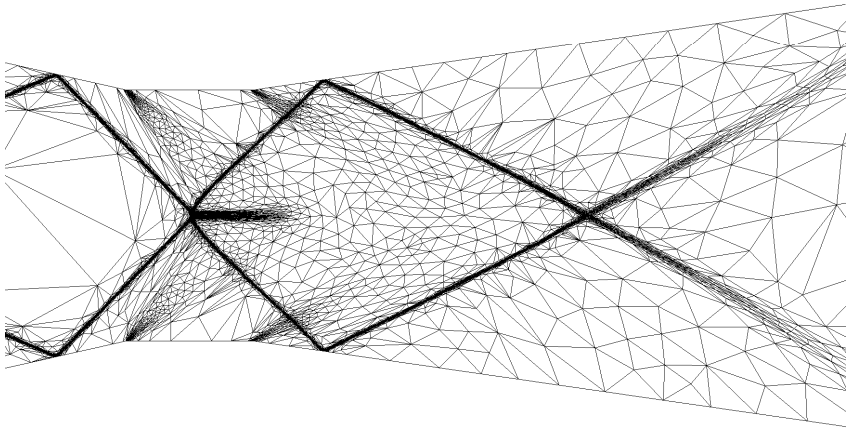
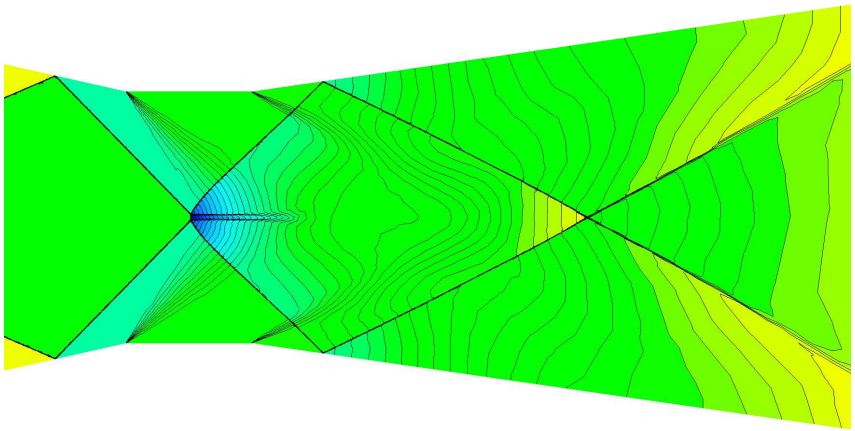
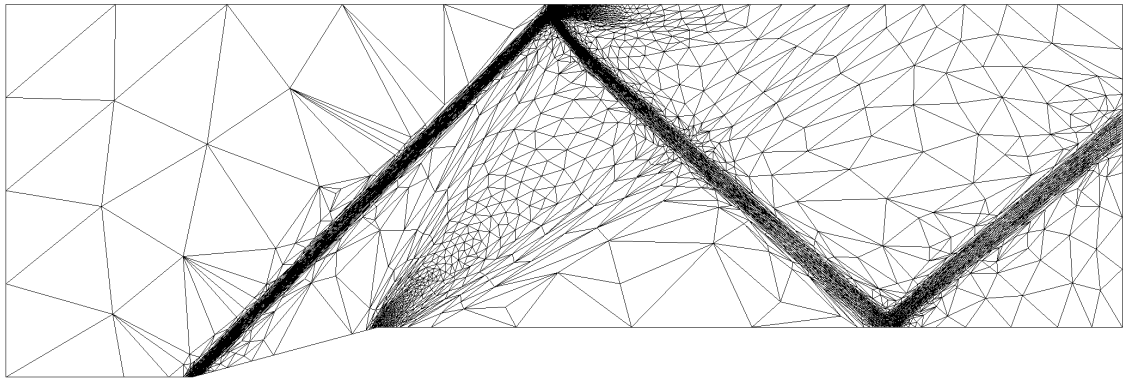
Earlier work on 2D anisatropic adaptation

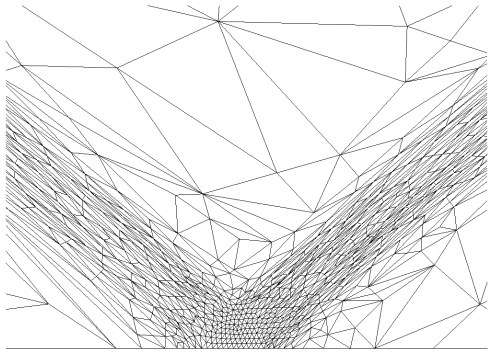




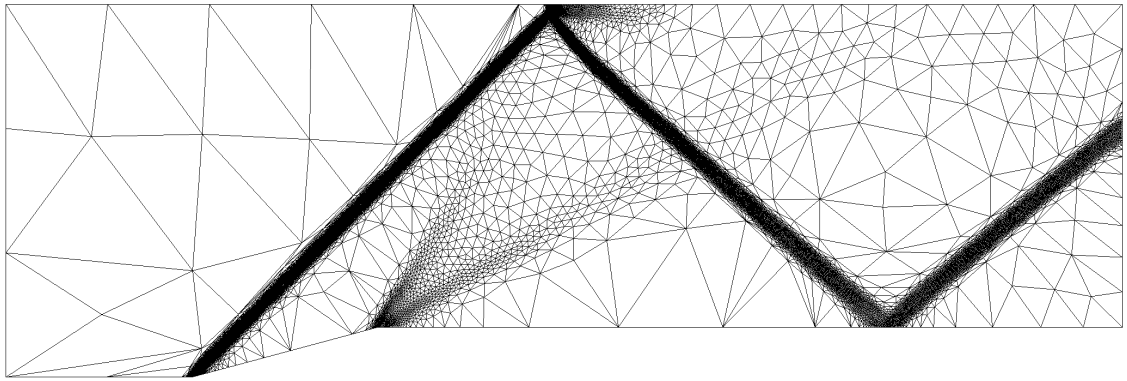


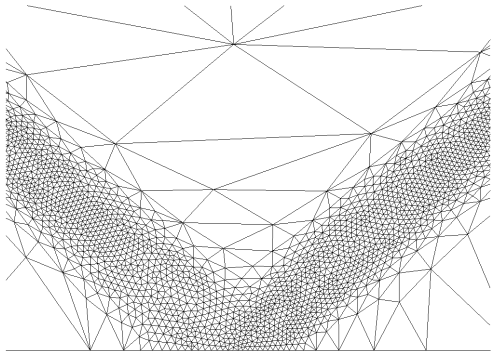
Earlier work on 2D anisatropic adaptation





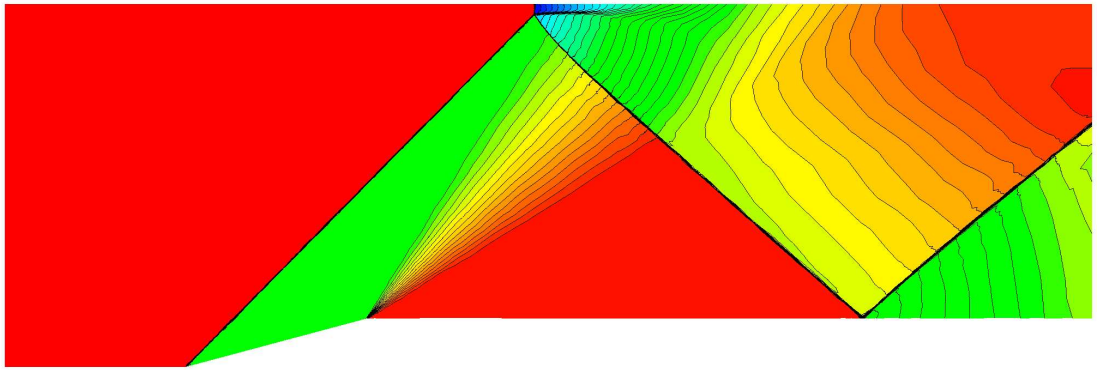
Anisotropic grid after 3 adaptations (6403 grid nodes)



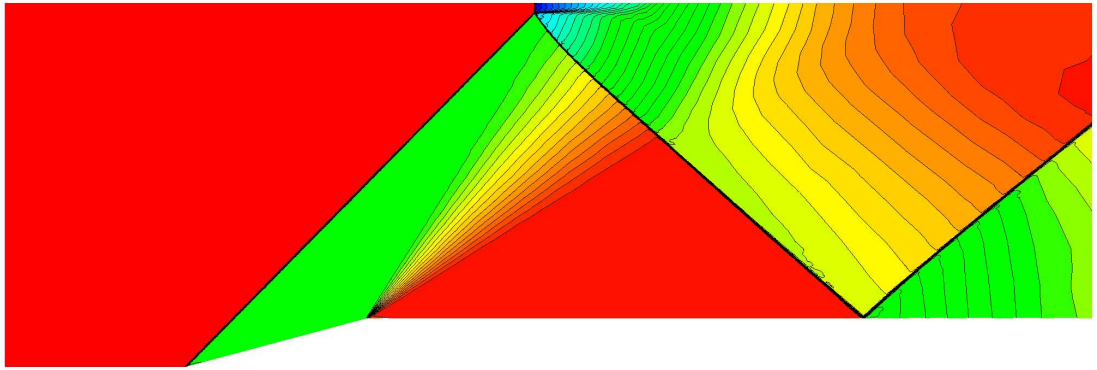


Isotropic grid after 3 adaptations (28078 grid nodes)

Earlier work on 2D anisatropic adaptation



Mach field calculated on the anisotropic grid



Mach field calculated on the isotropic grid

Plan of the presentation

- Anisotropic adaptation based on multidimensional interpolation error indicator
- Adaptation algorithm for unsteady flow problems
- Numerical examples
- Conclusion and future work

Anisotropic adaptation

- Based on grid regeneration which means that for every new adaptation step new grid is created
- Generation of the new grid is based on spacing defined by means of the metric field. The metric field is used to define a Riemann space in which the new grid cells have uniform spacing

  Castro-Daz, M.J., Hecht, F., Mohammadi, B., Pironneau., Anistropic Unstructured Mesh Adaptation for Flow Simulation, Int. J. Num. Meth. Fluids, 25, 4, 475-491, 1997

- Metric field is constructed according to the estimated error of the solution obtained on the old grid

Interpolation error

- Error estimator is based on the interpolation error
- For 2nd order solver the interpolation error is proportional to the Hessian of the current solution:

$$\mathcal{H} = \begin{bmatrix} \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 u}{\partial x\,\partial y} & \frac{\partial^2 u}{\partial x\,\partial z} \\ \frac{\partial^2 u}{\partial x\,\partial y} & \frac{\partial^2 u}{\partial y^2} & \frac{\partial^2 u}{\partial y\,\partial z} \\ \frac{\partial^2 u}{\partial x\,\partial z} & \frac{\partial^2 u}{\partial y\,\partial z} & \frac{\partial^2 u}{\partial z^2} \end{bmatrix} = R \cdot \Lambda \cdot R^{-1}$$

Calculation of the metric

- The metric field used for generation of the new grid is constructed from the decomposed Hessian:

$$\mathcal{M} = C^{-1} \quad |\mathcal{H}| = C^{-1}\, R \cdot \begin{bmatrix} |\lambda_1| & 0 & 0 \\ 0 & |\lambda_2| & 0 \\ 0 & 0 & |\lambda_3| \end{bmatrix} \cdot R^{-1}$$

- The constant C is calculated by equidistributing error calculated along all edges:

$$C = \frac{1}{n_e} \sum_i h_i^2\, \mathbf{w}_i^T |\mathcal{H}_i| \mathbf{w}_i$$

Limiting of the metric

- The final metric can be expressed as:

$$\hat{\mathcal{M}} = R \cdot \begin{bmatrix} \hat{\lambda}_1 & 0 & 0 \\ 0 & \hat{\lambda}_2 & 0 \\ 0 & 0 & \hat{\lambda}_3 \end{bmatrix} \cdot R^{-1}$$

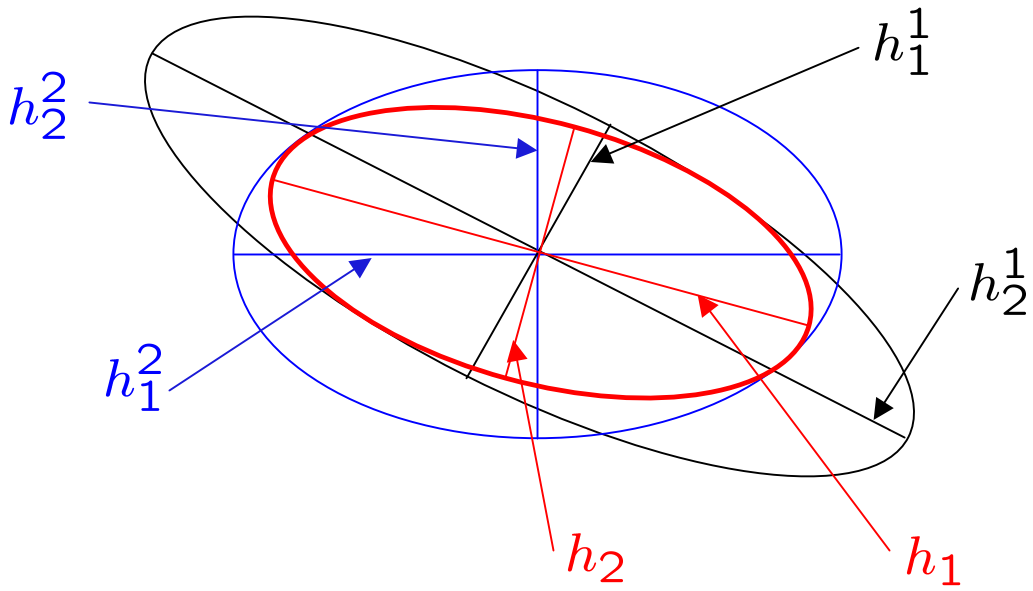
- The eigenvalues can be interpreted as $\lambda_i = 1/h_i^2$ where h_i stands for the length of i-th main axis of ellipsoid which should circumscribe the generated element

- In order to avoid unrealistic values the eigenvalues are modified:

$$\hat{\lambda}_i = \frac{1}{\max(h_{min}^2, \min(h_{max}^2, h_i^2))}$$

Intersection of metrics

In order to be able to adapt the grid with respect to many scalar fields it is necessary to combine many metric field corresponding to a scalar solution field (e.g. Mach number, entropy) into one metric field. It is done using intersection of metrics at each node.



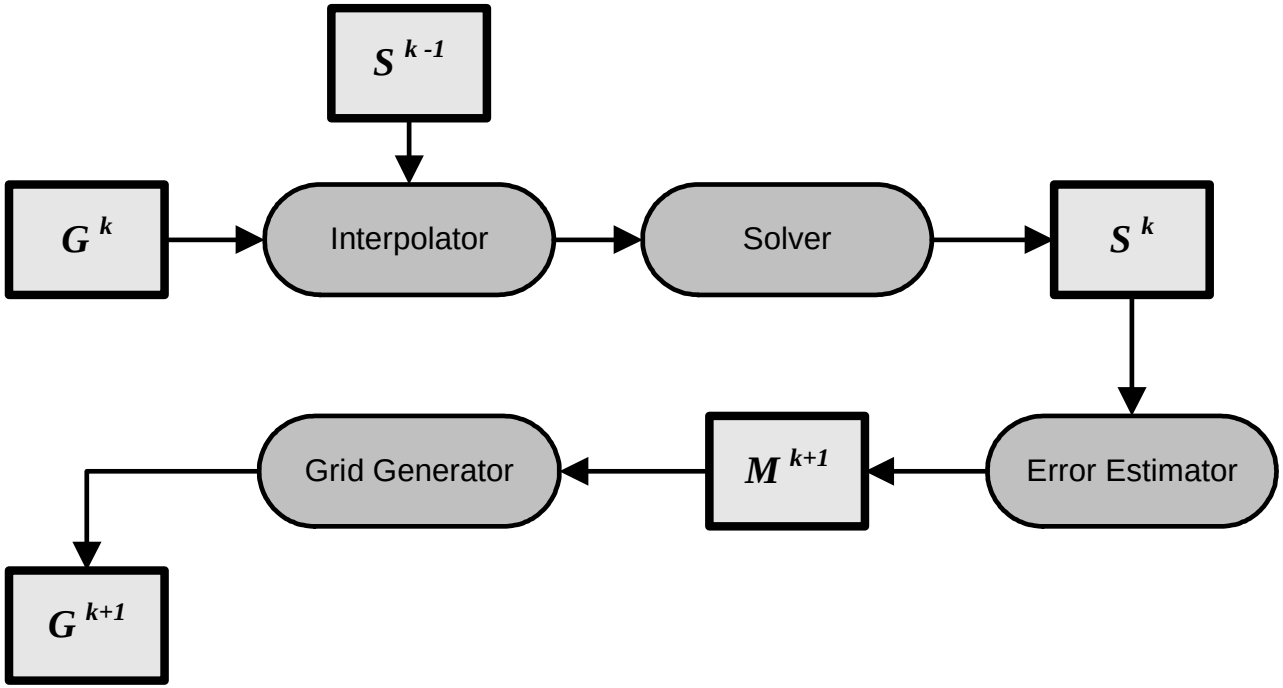
Reconstruction of the Hessian

- Based on Green formula:

$$\widetilde{\nabla u} = \frac{1}{\Omega} \oint_{\partial\Omega} u \, \mathbf{n} \, dS$$

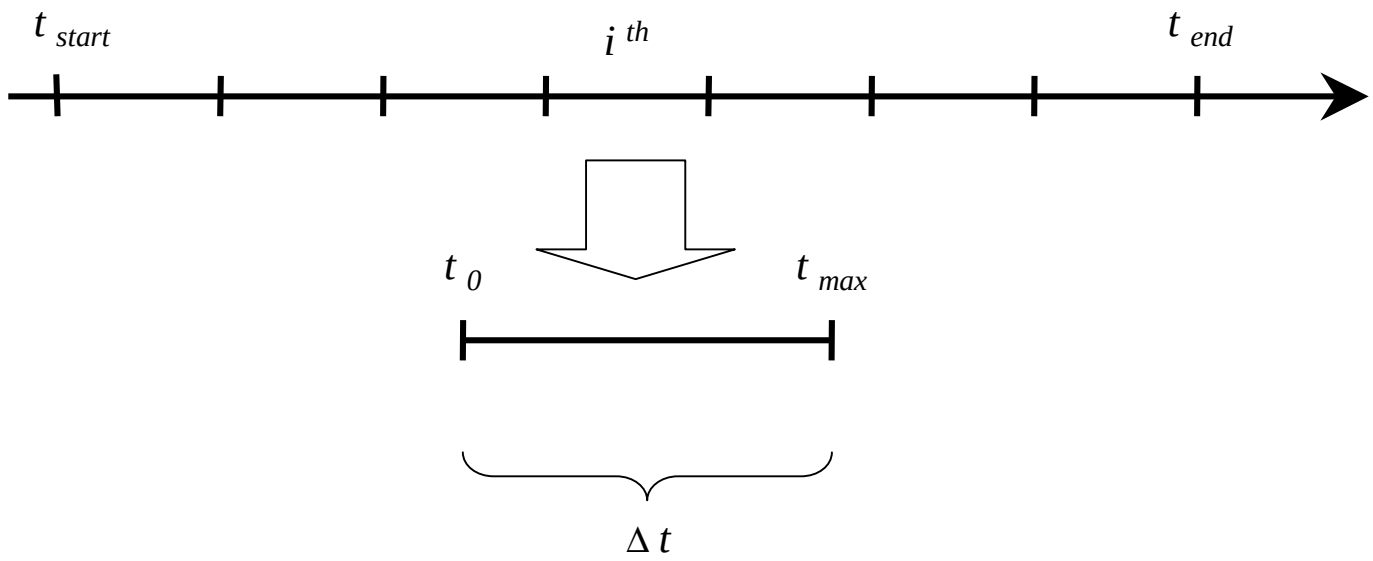
- It is done in two steps:
  - Calculate gradient
  - Calculate Hessian using the gradient

Adaptation algorithm for steady flows



G^k - grid after k adaptations
S^k - converged solution calculated on the G^k
M^k - metric field used for generation of G^k

Adaptation algorithm for unsteady flows

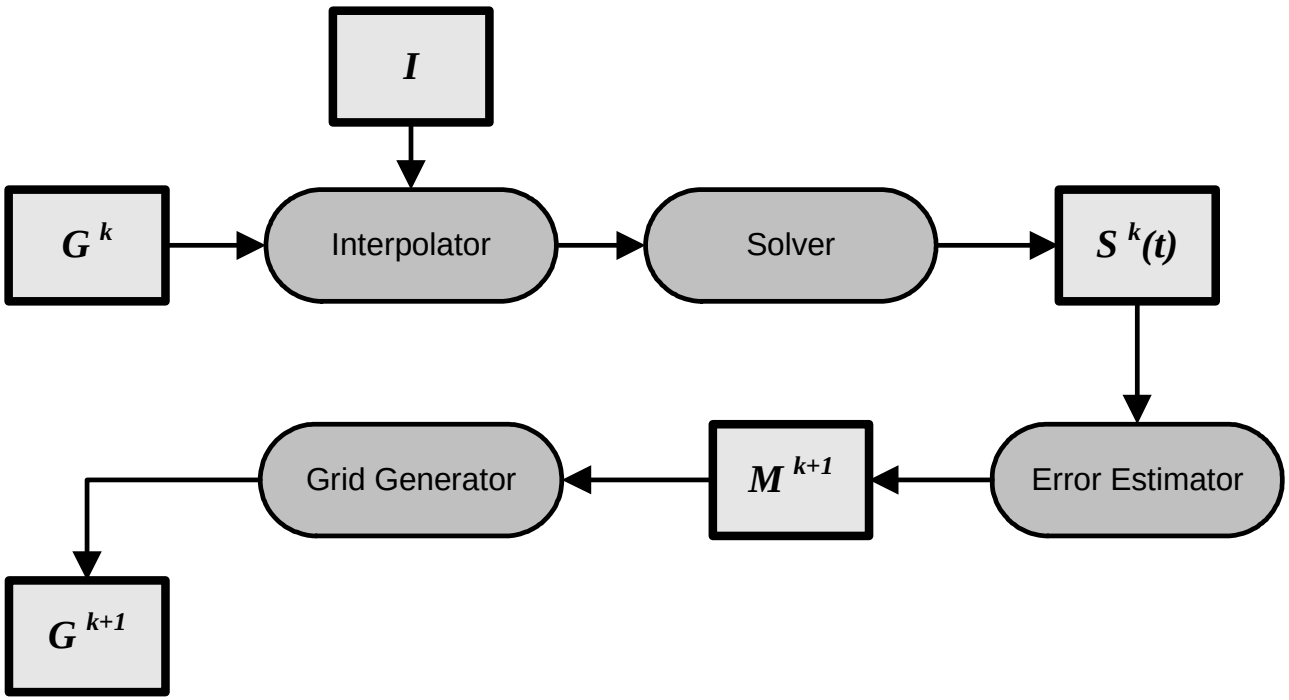


When solving unsteady flow the macro-step is splitted into sub-steps $[t_0, t_{max}]$ for which calculations are done on single adapted grid.

Alauzet, F., George, P.L., Mohammadi, B., Frey, P., Borouchaki, H., Transient fixed point-based unstructured mesh adaptation, Int. J. Numer. Meth. Fluids, 43, 729-745, 2003.

Adaptation algorithm for unsteady flows

Unsteady adaptation for the time sub-step - $t \in [t_0, t_{max}]$

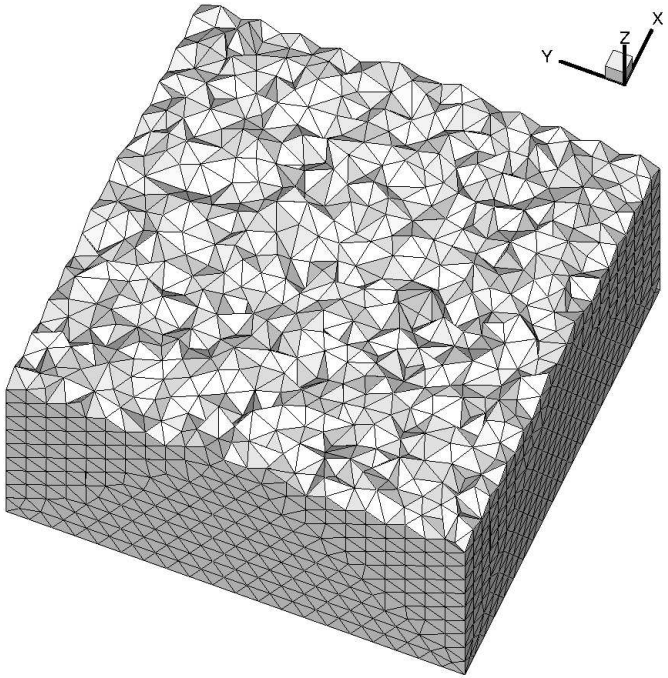


I - initial solution (the solution S^{kmax} from previous time sub-step is used)
G^k - grid after k adaptations
S^k - solution calculated on the G^k
M^k - metric field used for generation of G^k

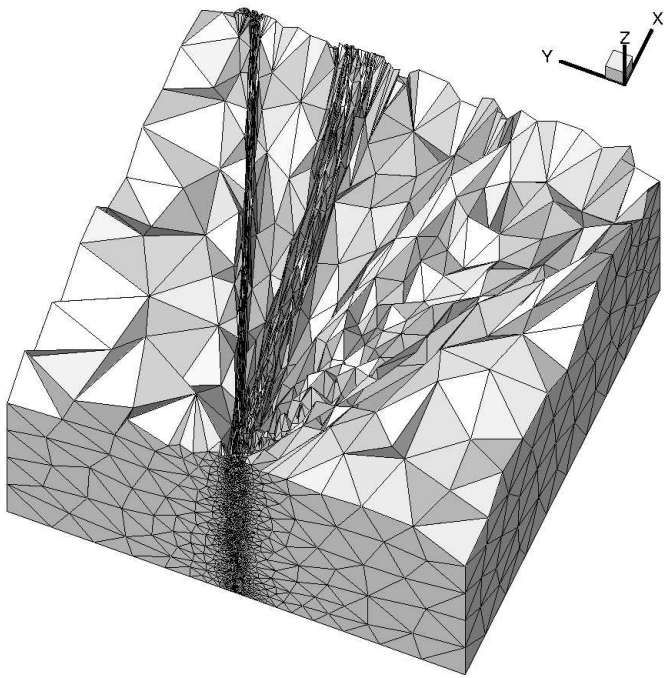
Two 3D jets flow inside a cube

- Steady flow with two jets coming into the cube domain. The solution should include shockwave, contact wave and expansion fan (Riemann problem).
- Euler equations
- Residual Distribution solver developed in VKI inside the COOLFluiD framework. First-order N-scheme was used.

Two 3D jets flow inside a cube

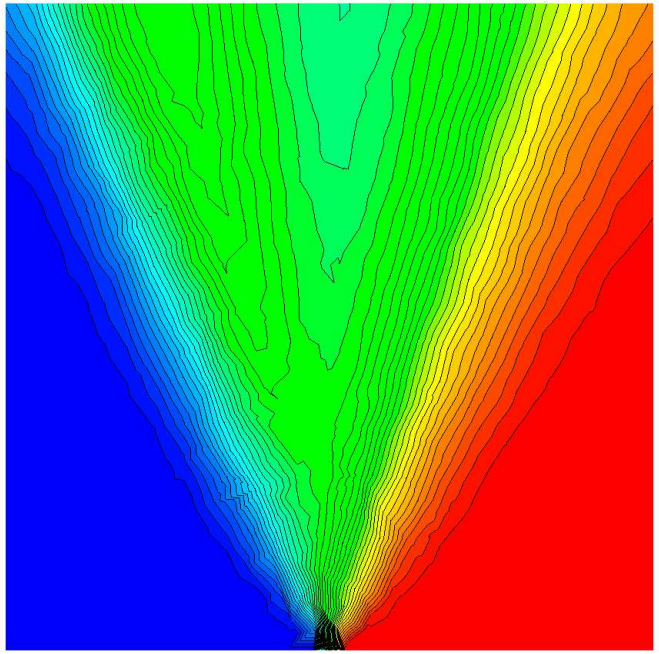


Initial uniform grid
(12618 nodes)

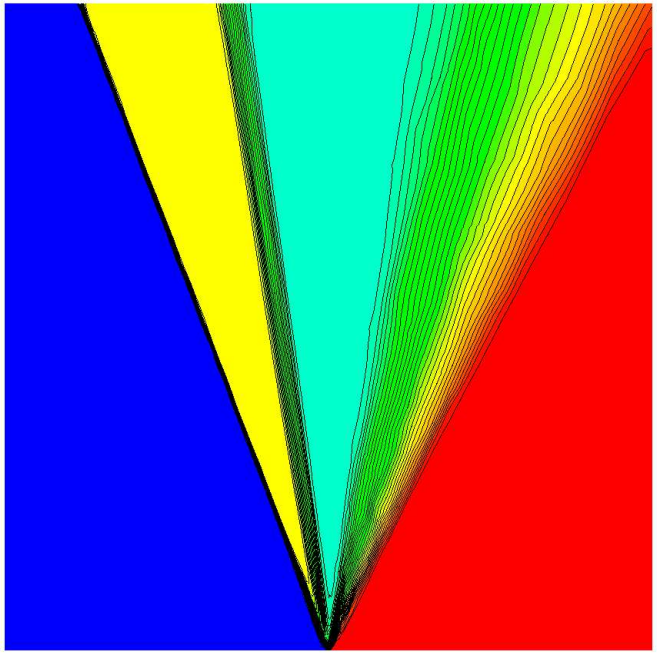


Grid after 8th level of adaptation
(47832 nodes)

Two 3D jets flow inside a cube



Density field computed on the initial grid. Solution shown on the plane in the middle of the cube.

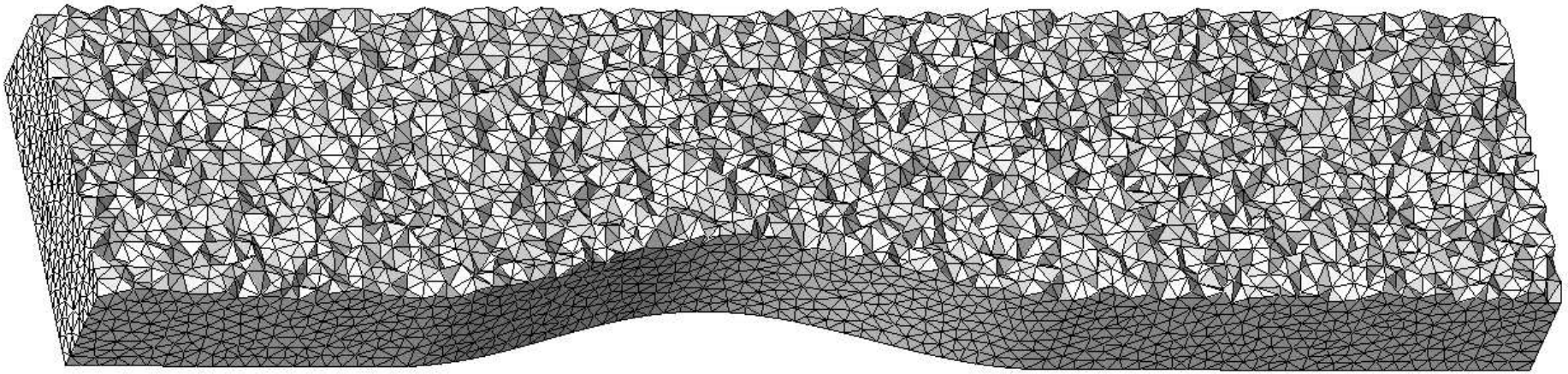


Density field computed on the grid after 8th level of adaptation. Solution shown on the plane in the middle of the cube.

3D flow in a channel with sinusoidal bump

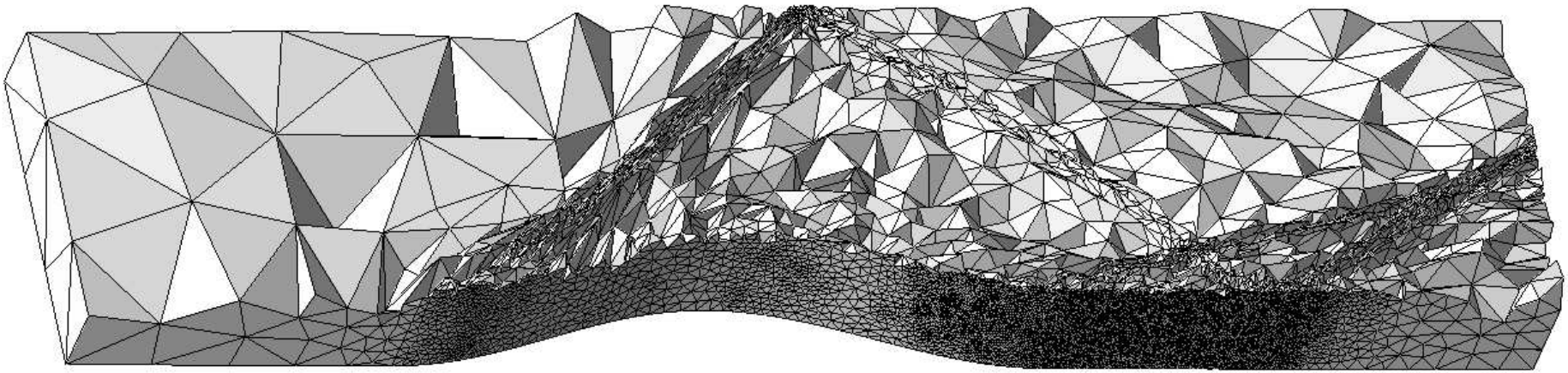
- Steady flow with supersonic inlet and fully conductive walls
- Ideal MHD equations
- Finite Volume Method solver developed in VKI within COOLFluiD framework

3D flow in a channel with sinusoidal bump



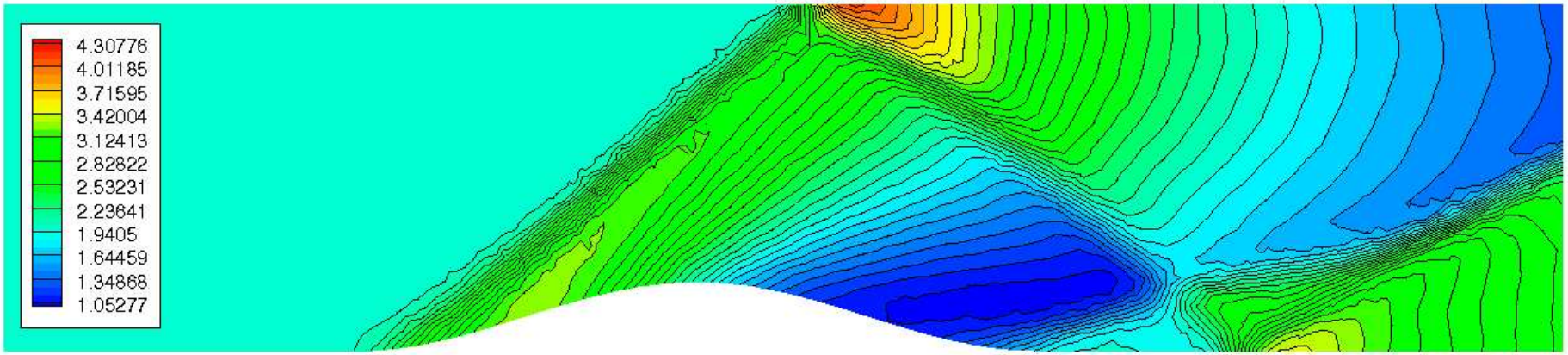
Initial uniform grid (23117 nodes)

3D flow in a channel with sinusoidal bump



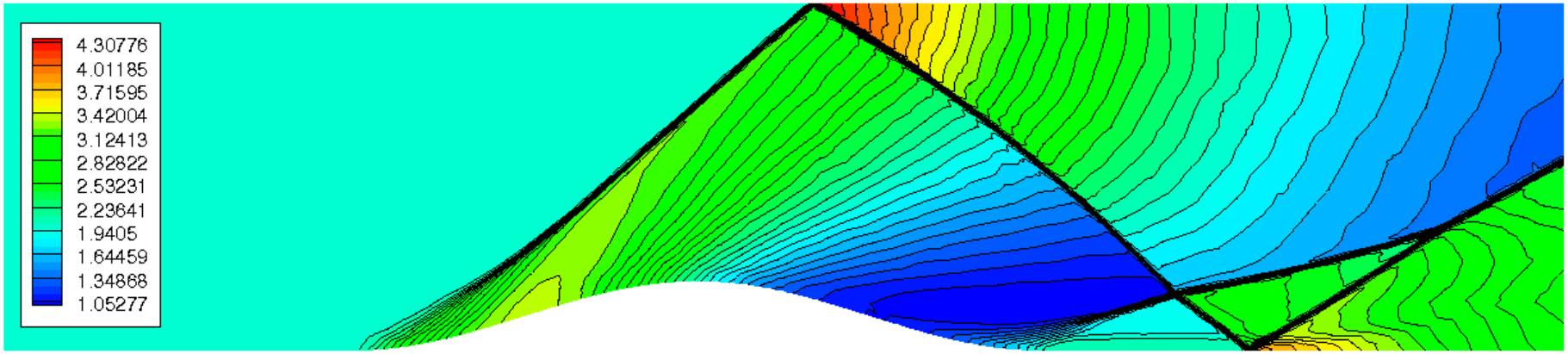
Grid after three levels of adaptation (36361 nodes)

3D flow in a channel with sinusoidal bump



Field of magnitude of the magnetic induction vector computed on the initial grid (at the plane in the middle of the channel)

3D flow in a channel with sinusoidal bump



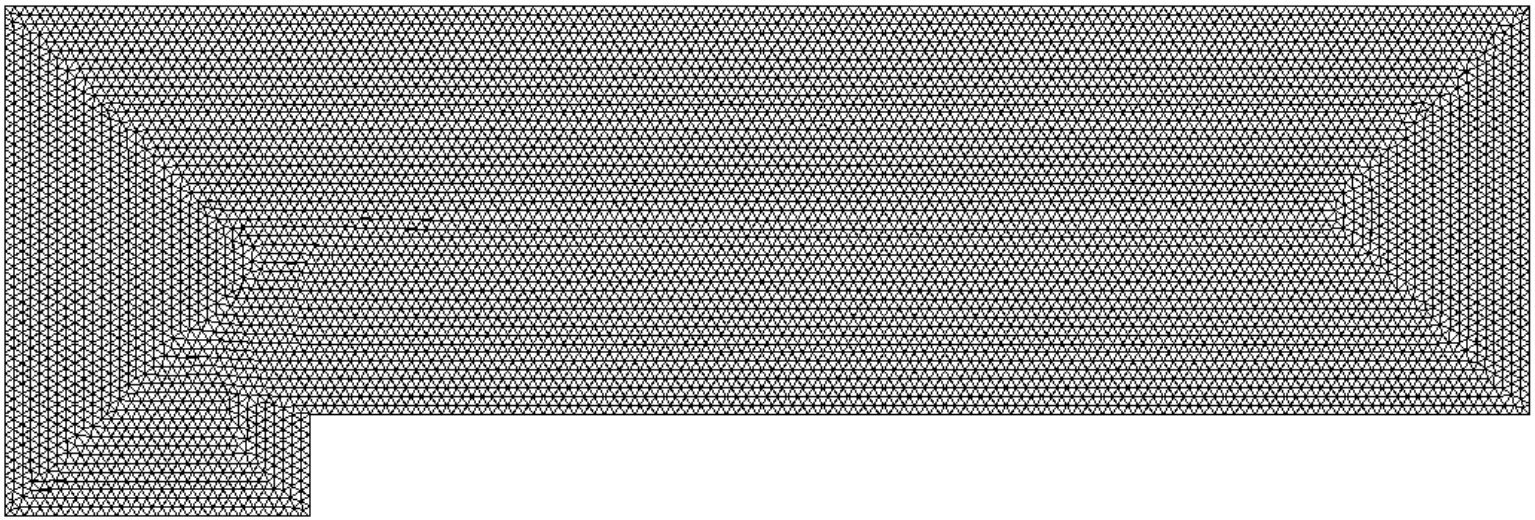
Magnitude of the magnetic induction vector computed on the grid
after three levels of adaptation
(at the cross-section in the middle of the channel)

2D flow in channel with forward facing step

- Unsteady flow with Mach number of 3.0 at inlet
- Euler equations
- Solver based on 2nd order Cell Centered Finite Volume Method with WENO reconstruction

2D flow in channel with forward facing step

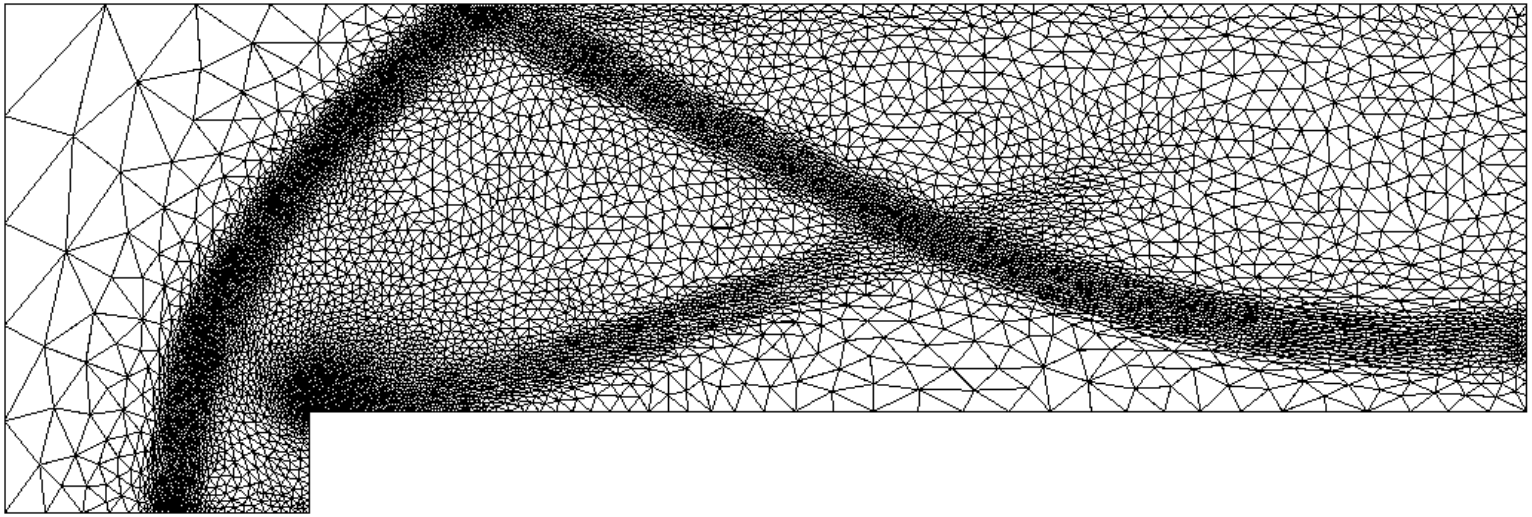
Sequence of grids used for calculations for time $t \in (0.98, 1.0]$



Uniform grid used for non-adaptive calculations (7444 nodes)

2D flow in channel with forward facing step

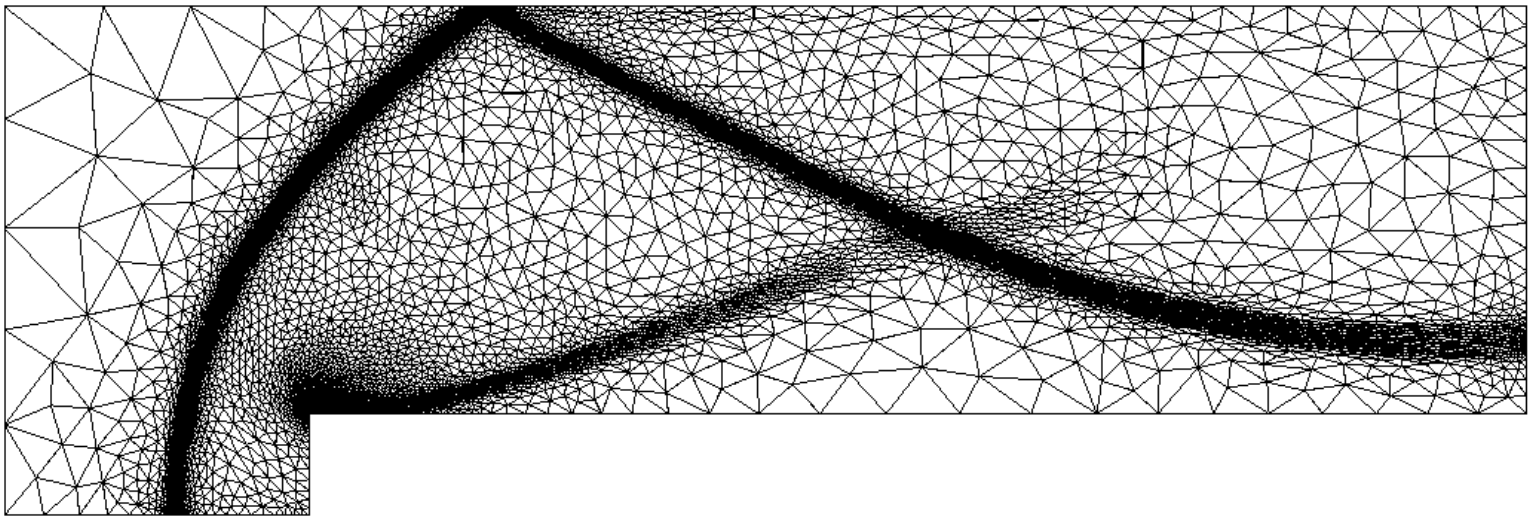
Sequence of grids used for calculations for time $t \in (0.98, 1.0]$



Grid created after one step of adaptation (8553 nodes)

2D flow in channel with forward facing step

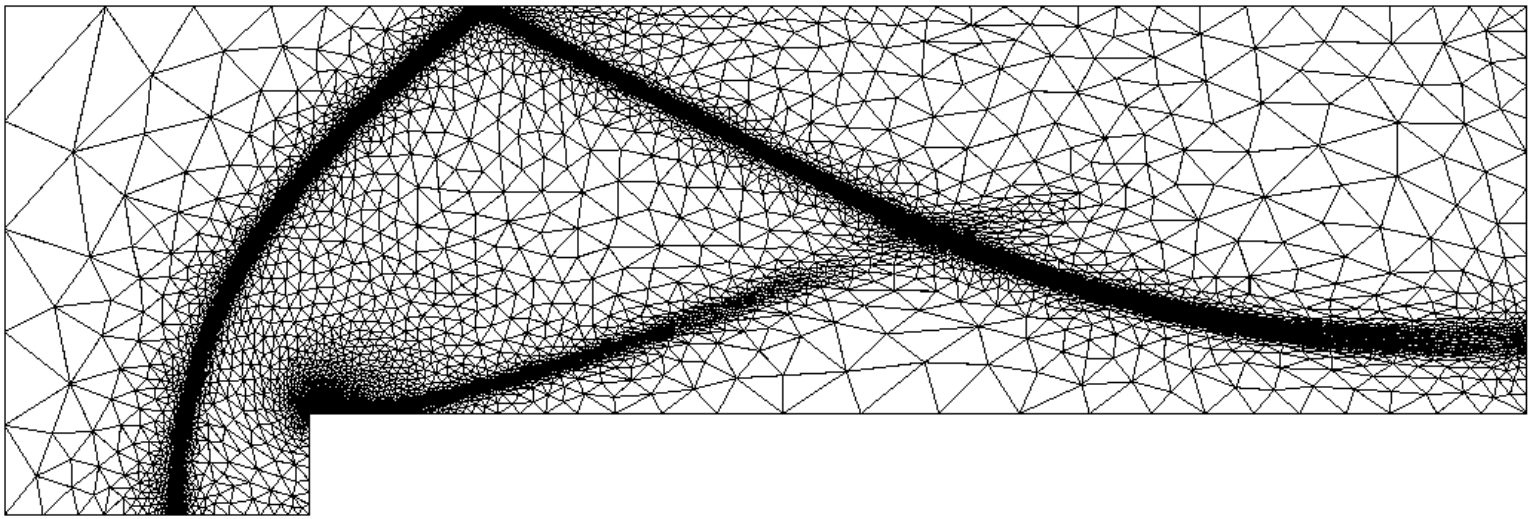
Sequence of grids used for calculations for time $t \in (0.98, 1.0]$



Grid created after two steps of adaptation (10227 nodes)

2D flow in channel with forward facing step

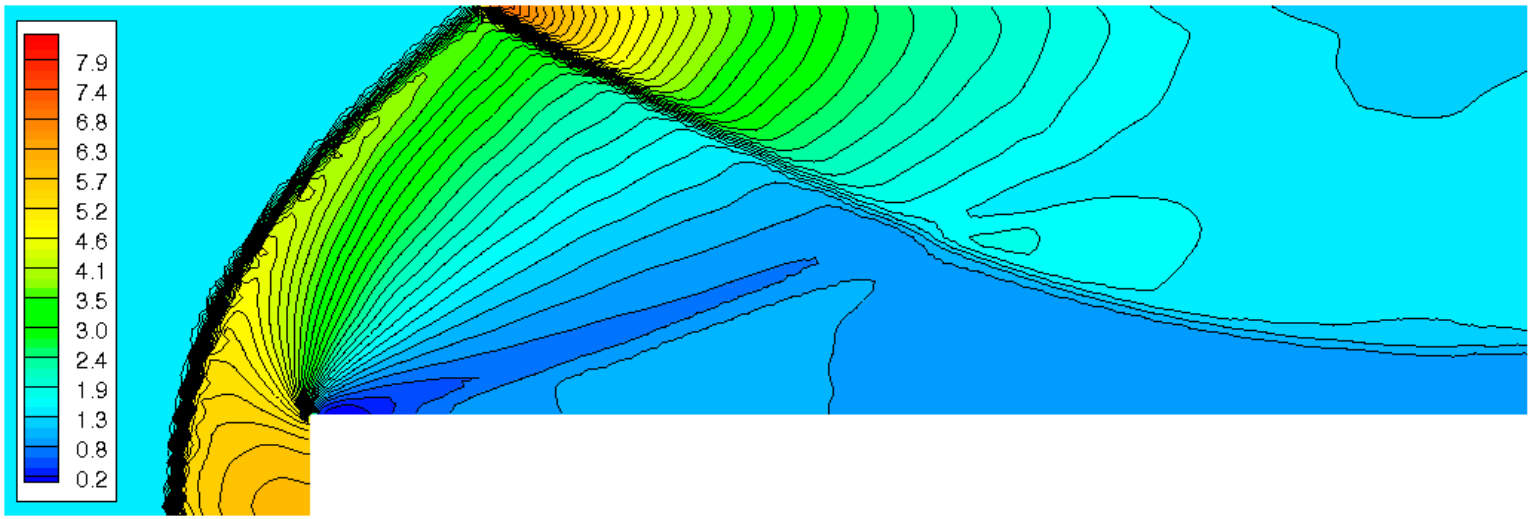
Sequence of grids used for calculations for time $t \in (0.98, 1.0]$



Grid created after three steps of adaptation (22168 nodes)

2D flow in channel with forward facing step

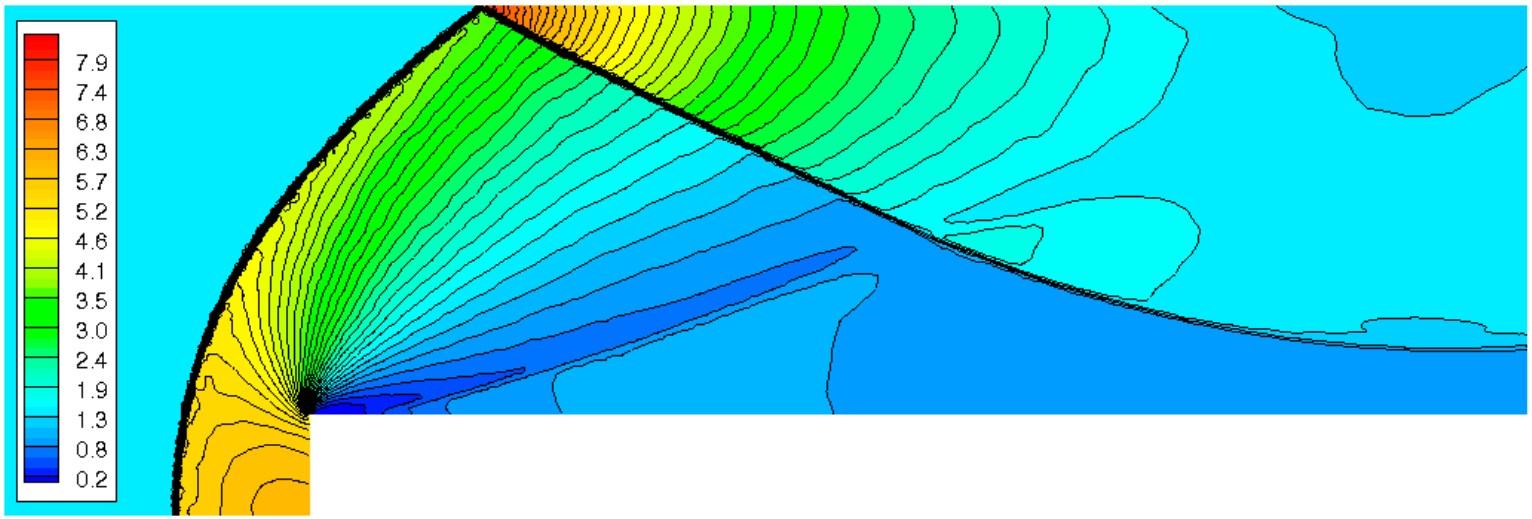
Solution for different levels of adaptation for time $t = 1.0$



Density field computed on the un-adapted grid

2D flow in channel with forward facing step

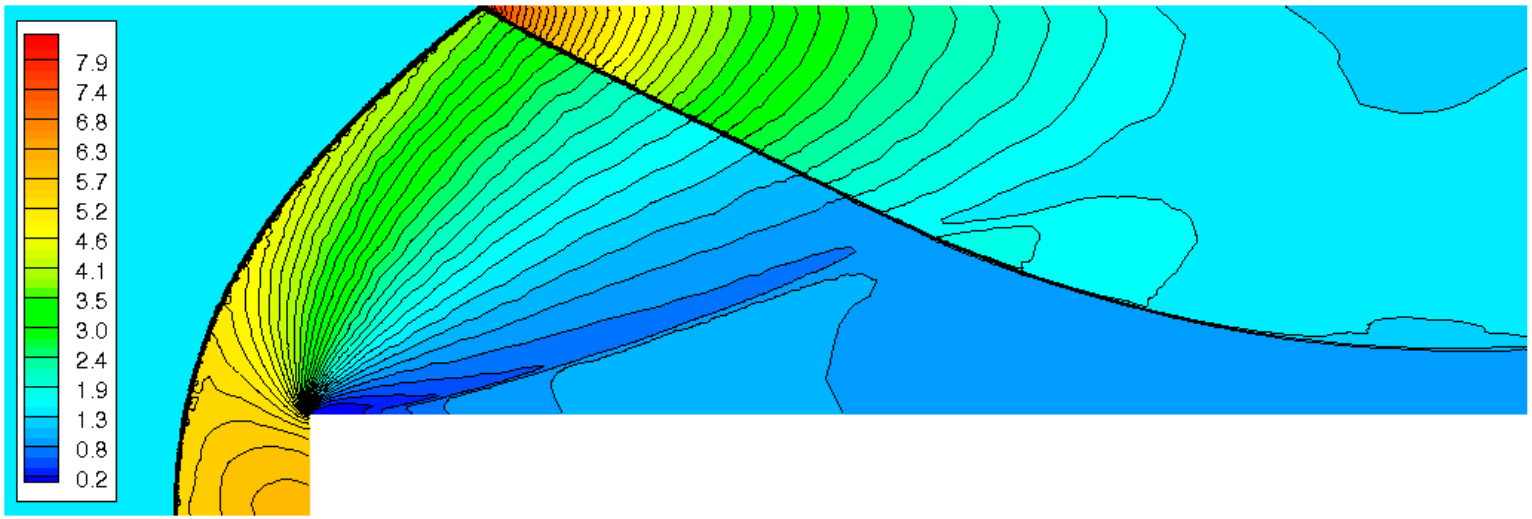
Solution for different levels of adaptation for time $t = 1.0$



Density field computed on the grid after one level of adaptation

2D flow in channel with forward facing step

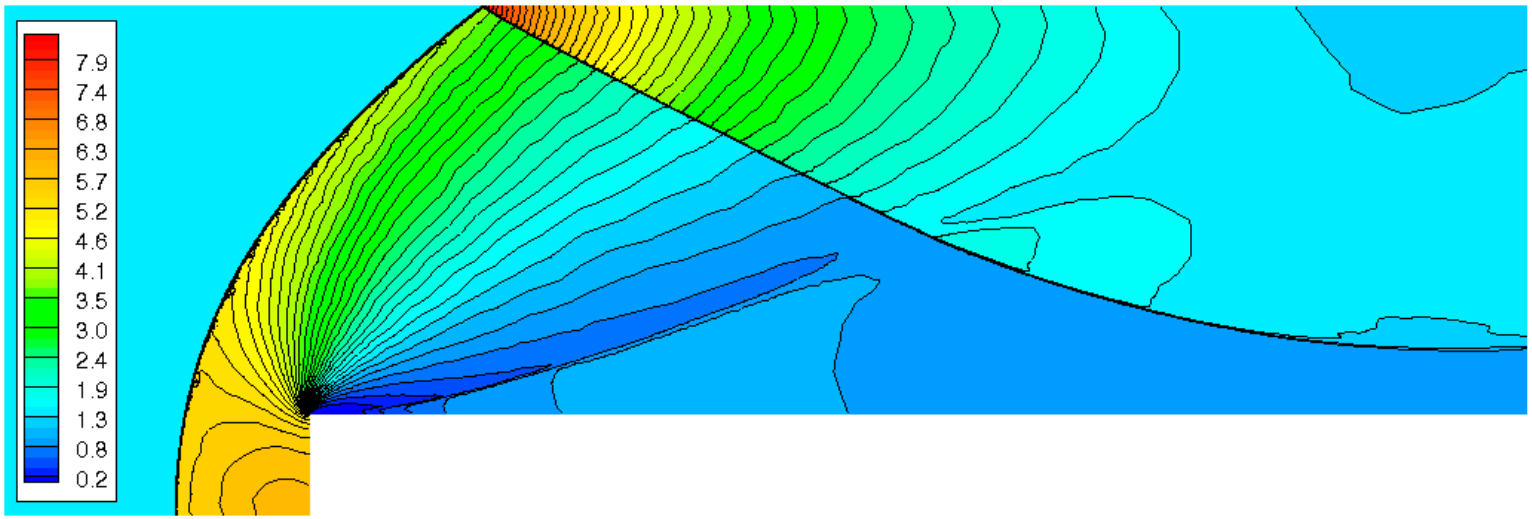
Solution for different levels of adaptation for time $t = 1.0$



Density field computed on the grid after two levels of adaptation

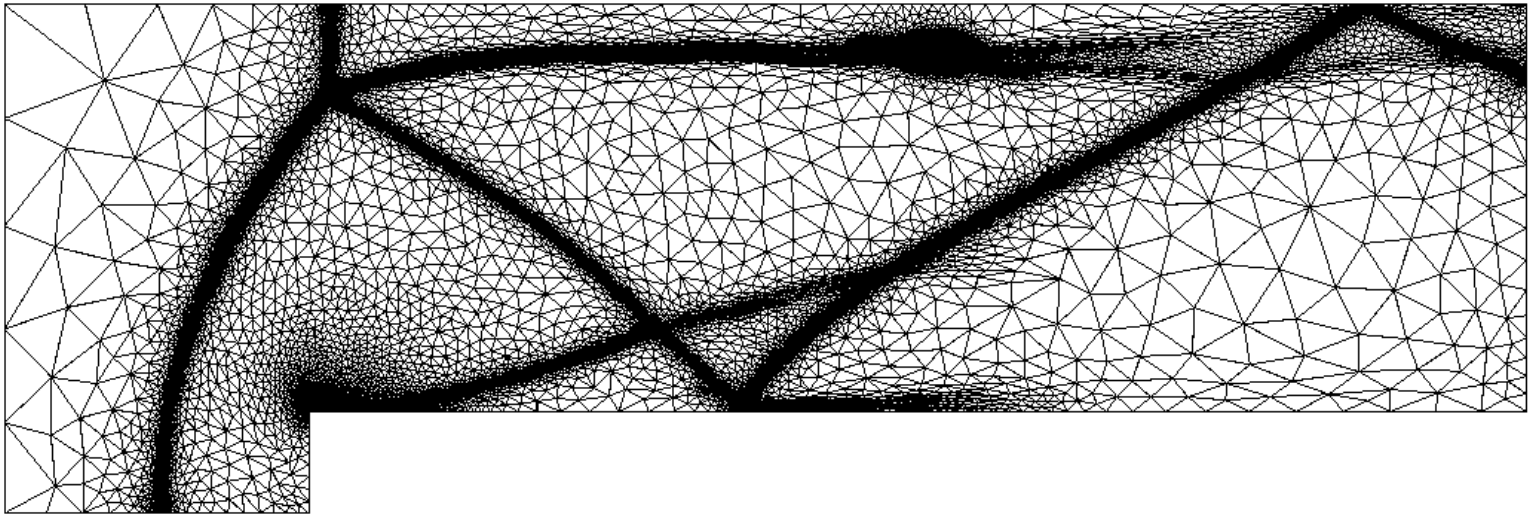
2D flow in channel with forward facing step

Solution for different levels of adaptation for time $t = 1.0$



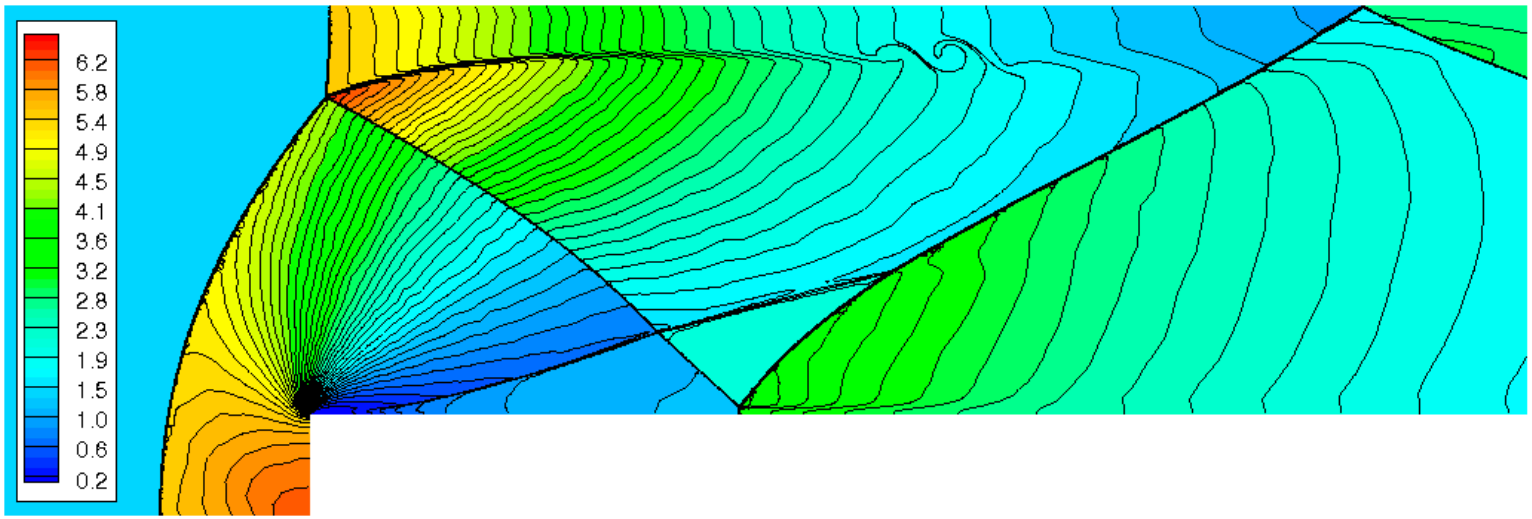
Density field computed on the grid after three levels of adaptation

2D flow in channel with forward facing step



Grid (42193 nodes) after three levels of adaptation used for calculations for time $t \in (2.98, 3.0]$

2D flow in channel with forward facing step

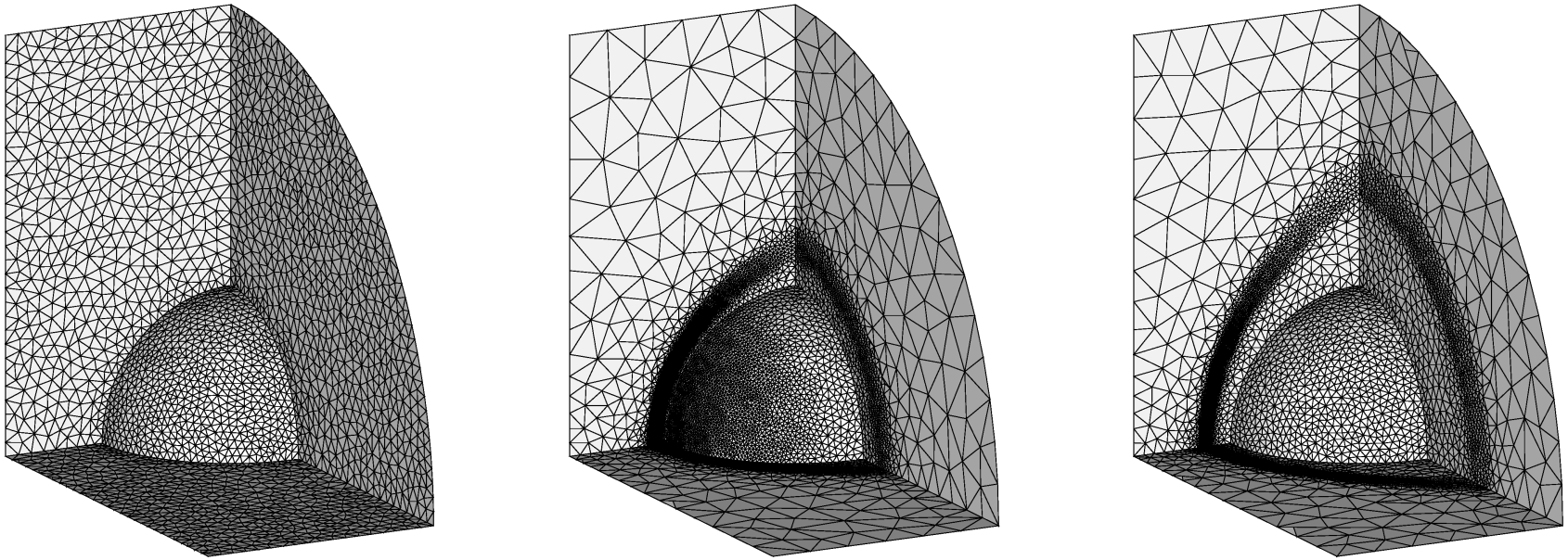


Density field computed on the grid after three levels of adaptation
(time t = 3.0)

3D unsteady flow past sphere

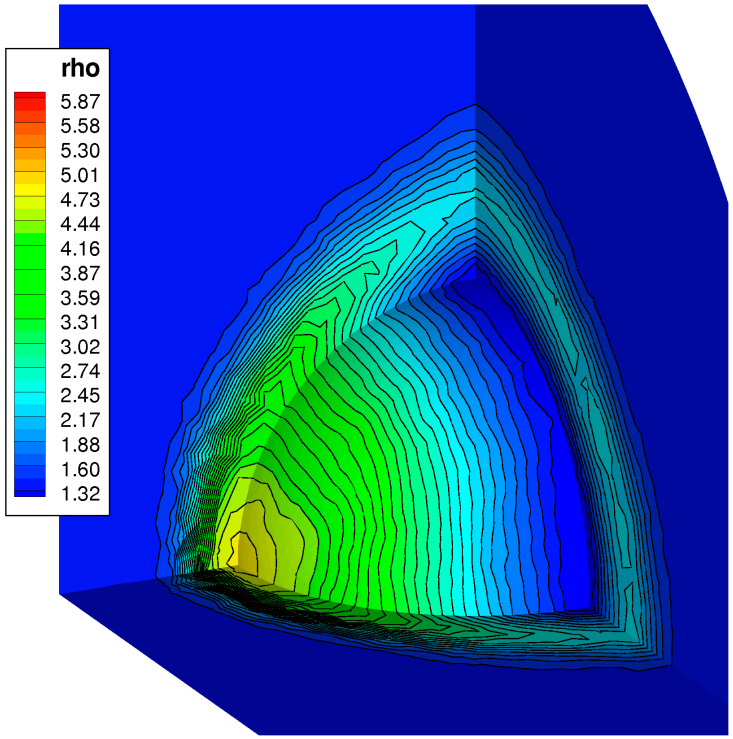
- Unsteady flow with Mach number of 3.0 at infinity
- Euler equations
- Finite Volume Method solver developed in VKI within COOLFluiD framework

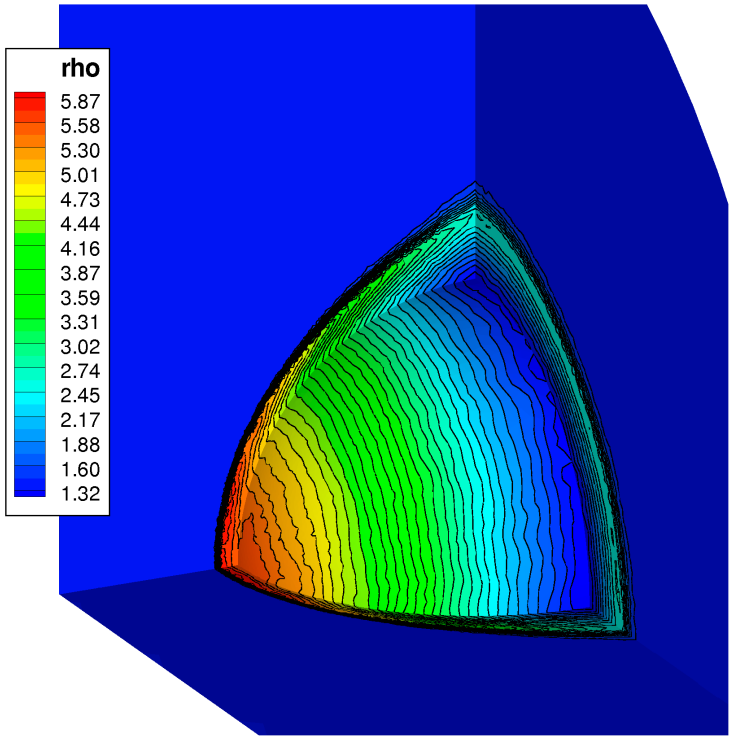
3D unsteady flow past sphere



Initial grid and grids after 2 adaptations for t=0.003 and t=0.019
(number of nodes:18280, 80107, 54184 respectively)

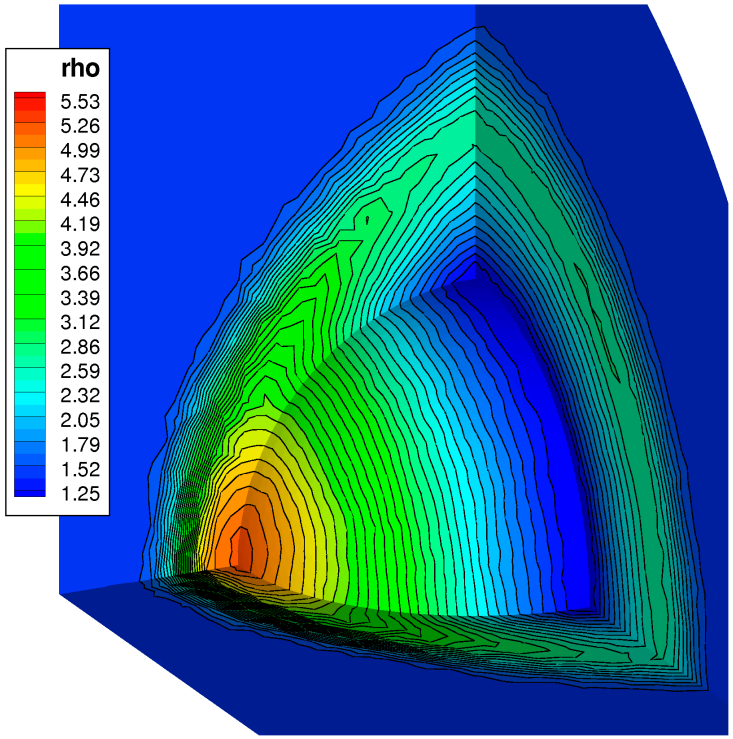
3D unsteady flow past sphere

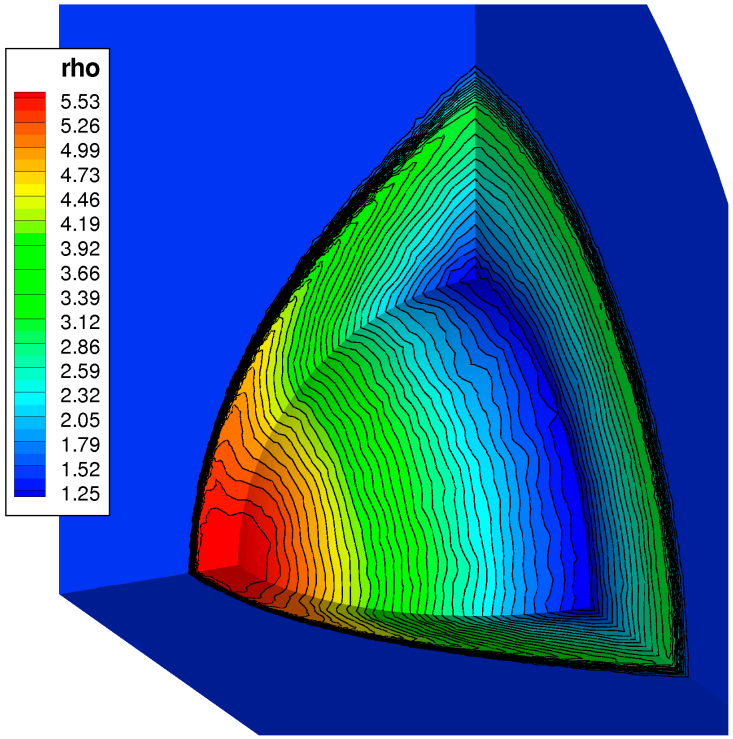




The density for time t = 0.003 calculated on the initial grid and on the grid after two adaptations

3D unsteady flow past sphere





The density for time t = 0.019 calculated on the initial grid and on the grid after two adaptations

Conclusions

- The presented algorithms is capable of 2D and 3D unsteady anisotropic adaptation
- It has been shown that the algorithm can significantly improve results of calculations for Euler problems
- The error estimator based on the Hessian of the solution works fine even if the assumption that the solver is 2nd order is not valid (however the grid created for such cases may not be optimal)

Future work

- Improvement of the 3D anisotropic grid generator
- Correction of the metric field in order to improve anisotropy for areas along strong shocks
- Modification of the error estimator to be fully compliant with higher order schemes

Ackowledgements

The presented work was supported
by Belgian Federal Science Policy Office