ABSTRACT

Due to successful noise reduction strategies concerning fan- and jet-noise in gas turbine configurations, the relevance of combustion noise is increasing. In order to distinguish between turbulent noise and combustion noise a model gas turbine combustor consisting of a swirl burner and an exit nozzle of Laval-shape is investigated. Because of the instationary character of the flow this configuration is investigated by means of Large Eddy Simulation (LES). Numerical results are first validated by comparison with experimental data. Then a numerical study of noise generated by turbulent flow instabilities is carried out. Providing an extensive temporal and spatial analysis of the isothermal flow length- and timescales as well as vorticities are investigated with regard to the formation of rotating flow-instabilities in the recirculating swirling flow. Subsequently noise sources are identified and evaluated based on the Lamb vector consideration. It appears that the noise sources increase with an increasing swirl number ratio.

1 INTRODUCTION

In many modern gas turbines, one of the problems to be faced is the acoustic oscillations in the combustion chamber. With regard to the increasing interest in environmental pollution, this phenomenon in aero-engines has especially received a lot of attention. Thus, there is a need for improved prediction methodologies that may further contribute to the understanding of the noise generation mechanisms, assisting the design process of aircraft engines with low noise emissions. As pointed out in [19], a comprehensive understanding of this phenomenon may most likely be possible by the use of numerical simulations. These may provide any type of information needed for the analysis of the investigated mechanism. This work is focused on noise generated inside a combustion chamber system. In this framework, the total noise radiated can be subdivided in a direct and indirect part like shown by a generalized acoustic energy equation by Dowling [2]. The direct noise sources are related to the unsteady combustion process itself, e.g. to unsteady heat release. The indirect (combustion) noise is generated when fluid with a nonuniform entropy distribution is accelerated in, or convected way through, the nozzle located at the downstream end of the combustion chamber. The accelerated or decelerated hot spot radiates sound due to a fluctuating mass flux. In gas turbines, the inlet guide vanes for the first turbine stage serve as...
nozzle for the combustion chamber. The flow in this nozzle is choked in aero-engines in practically all relevant operating conditions. The underlying theory of these acoustic mechanisms is described by Marble and Candel [13]. The separation of hydrodynamic, entropy and combustion noise was investigated earlier via cross spectral analysis [16]. Although there have been numerous experimental and numerical studies to date, the actual contribution of both parts in the total noise emission of aero-engine combustors is still not well understood. In order to evaluate the respective contribution of turbulent flow generated noise and unsteady combustion process generated noise, a model gas turbine combustor is investigated.

Since noise generation inside the combustor is an inherently unsteady process, LES appears as the most suitable computational tool while DNS of such a high-Reynolds number flow would result in tremendous resolution requirements that are far beyond the capability of the fastest supercomputers available today. For such multi-scale problems that involve a wide range of length and time scales, the use of the classical U-RANS remains limited as all relevant scales of turbulence are modeled. New approaches including advanced turbulence models capable of accurately modeling a wide range of turbulence scales are developed, and may be promising [21]. A recent overview of the applications of LES to noise prediction can be found in [19]. Besides the effect of SGS-modeling on the prediction of the high-frequency portion of the noise spectra, in general, the LES results in the literature confirm the potential of LES to predict jet noise.

In this work we aim at capturing hydrodynamic instabilities, as well as the location and structures of the sound sources. In a first step it is necessary to determine the noise contribution as a consequence of turbulent flow instabilities. For this purpose, we concentrate on the isothermal turbulent flow case. It is characterized by a back-flow process and swirl phenomena leading to strong coherent velocity fluctuations along with flow instabilities. The flow structure characteristics appearing are evaluated based on an extensive temporal and spatial analysis of the flow. Due to the presence of a rotating flow instability, it is essential to segregate the coherent motion from the turbulent one. At different effective swirl intensities, the influence of the swirl number on the Strouhal number and the turbulent scales is pointed out. The mechanism of an induced secondary flow instability in the convergent-divergent outlet nozzle is further investigated. The instability behavior of a ring jet with back-flow and swirl has been investigated analytically by Michalke [14] on the basis of the inviscid linearized theory for incompressible flow. Using a modified "Monkewitz profile" [15] for the flow field the frequency and the growth rate for the first azimuthal mode were predicted for different profile parameters like swirl intensity and back-flow ratio. The result of this instability analysis will be compared with the results of the present LES calculations.

To perform this task, an appropriate 3D-LES code is applied. The experimental setup employed is outlined in section 2. Section 3 covers the modeling and numerical procedure completed by the numerical configuration and boundary conditions in section 4. To validate the code a comparison between experimental data, analytical values and numerical results for the flowfield and the energy spectra is presented and discussed in section 5. The flow analysis through a study of the coherent structure formation, the large-scale vortices and the secondary flow-instability inside the Laval-nozzle is carried out in section 6. An investigation of sensitivity effects of some operating conditions on noise production follows in section 7. The essential results of this work are finally summarized in section 8.

2 EXPERIMENTAL SETUP

In order to match the characteristic flow field of a realistic swirl stabilized gas turbine combustor the setup for the experimental investigation is carefully chosen. The main goal of the developed model is to capture a swirling flow through an area of large transitions of the cross-section. Therefore the axisymmetric configuration consists of a swirl burner, an exit nozzle of Laval-shape and an exhaust duct to model turbine guide vanes following the gas turbine combustor as sketched in figure 1.

![Figure 1. Sketch of the model combustion chamber setup used for the experiments.](image)

The swirl is generated in an inner and an outer co-rotating air-flow which are both fed from a common plenum chamber shown in figure 2. The inner air nozzle has a diameter of 15.4 mm while the outer annular air nozzle has 17 and 25 mm inner and outer diameter, respectively. For thermal and mixing investigations gas can be injected as fuel through an annular slot between the two air streams. In the experiments the air flow at room temperature with a Reynolds-number of Re ≈ 13,350 based on the outer nozzle diameter and the bulk velocity at the nozzle exit, $U = 8.31 \text{ m/s}$ have been considered.

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The cylindrical fused quartz glass combustion chamber has an inner diameter of 100 mm and a length of 1.13 mm with 3 mm wall thickness. A convergent–divergent outlet nozzle shown in figure 1 connects the combustion chamber with an exhaust tube of 100 mm diameter. The used throat diameter of $D_{\text{min}} = 17$ mm leads to a Mach number of $Ma = 0.05$ in the isothermal case.

At the operating point of the subsequent investigations the mass flow rate of air amounts to $13$ Nm$^3$/h which corresponds in the thermal case including $1$ Nm$^3$/h of methane gas to a thermal capacity of $10$ kW with an equivalence ratio of $0.726$ and a nominal swirl number of $S_n = 0.55$.

The velocity measurements were conducted using a commercial LDV system (DANTEC) and an Ar-Ion laser. The optical configuration consists of a two-component setup ($\lambda_{\text{laser}} = 488$ and $514$ nm). As seeding particles $\text{ZrO}_2$ with a particle size up to $10$ $\mu$m were used.

Performing LDV measurements in objects with curved surfaces demands certain attention to refraction effects of the laser beams. Due to the cylindrical shape of the combustion chamber the effects are different for the two components of the LDV. Necessary corrections are described in detail by Fritz [6]. However, for the measurements along the optical axis of the LDV setup only the axial velocity component was acquired. Two-component measurements were performed at fixed radial positions along axial lines. In order to analyze the photomultiplier signals two DANTEC burst spectrum analyzers (BSA) were used. In the case of two-component measurements coincident signals were evaluated which enable the extraction of cross-correlations ($<U'W'>$).

All following notations are based on the underlying coordinate system plotted in figure 1 where the origin is fixed in the outlet plane of the swirl burner nozzle on the center axis of the combustion chamber.

3 MODELLING AND NUMERICAL SETUP

3.1 Governing Equations

Since the flow velocities in most combustion applications are relatively low, the density can be assumed to be independent of the pressure. This yields an incompressible formulation and the low-Mach number approximation of the governing equations can be utilized. Still, the density varies strongly due to chemical reactions. The following two equations represent the Favre-filtered (density-weighted filtering: $\bar{\rho} \bar{f} = \bar{\rho} \bar{f}$) equations for mass and momentum, subjected to density variations.

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \bar{u}_i)}{\partial x_i} = 0$$

(1)

$$\frac{\partial \bar{\rho} \bar{u}_i}{\partial t} + \frac{\partial (\bar{\rho} \bar{u}_i \bar{u}_j)}{\partial x_j} = \frac{\partial \bar{\rho} \bar{u}_i}{\partial x_i}$$

(2)

In the case of non-premixed combustion, usually a fast chemistry approach is sufficient. Furthermore, the heat loss due to radiation can be neglected in most technical applications, especially in gas turbine combustors. Therefore the complete thermochemical state of the fluid is governed by the mixture of fuel and oxidizer, which is described by means of the mixture fraction $f$. The deduction of this approach is known as the “Shvab-Zel'dovich” formalism. For pure fuel, $f$ takes the value of $f = 1$ and for pure oxidiser the value of $f = 0$. The mixture fraction can be interpreted as a dimensionless fuel concentration or enthalpy. The filtered transport equation for the mixture fraction reads as follows.

$$\frac{\partial (\bar{\rho} \bar{f})}{\partial t} + \frac{\partial (\bar{\rho} \bar{f} \bar{u}_i)}{\partial x_i} = \frac{\partial \bar{\rho} \bar{D}}{\partial x_i} \left( \frac{\partial \bar{f}}{\partial x_i} + \bar{\rho} \bar{J}^{sgs}_{i} \right)$$

(3)

By filtering the transport equations for momentum and mixture fraction, two unresolved terms arise in the resulting equations (2) and (3) due to the non-linearities of the convective terms. These are namely $\tau^{sgs}_{ij}$, the sub-grid scale stresses, and $\bar{J}^{sgs}_{i}$, the sub-grid scale scalar flux vector, respectively. The sub-grid scale stresses are closed by the standard Smagorinsky model [17]. Hereby, the anisotropic part of the sub-grid scale stress tensor is modelled with an eddy-viscosity assumption, while the isotropic part is included into the pressure term.

$$\tau^{sgs}_{ij} - \frac{1}{3} \bar{\tau}^{sgs}_{kk} \delta_{ij} \approx 2 \nu_t \bar{S}_{ij}$$

with $\nu_t = (C_S \Delta)^2 |\bar{S}_{ij}|$

(4)
\[ \tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \] (5)

The model coefficient \( C_S \) in equation (4) is obtained by the dynamic procedure – originally proposed by Germano [7] – using the least-squares approach following Lilly [12].

The filtering operation is performed implicitly by means of the finite volume discretization. Therefore, the filter width \( \Delta \) coincides with the size of the grid cells. Furthermore, no special wall treatment is included into the sub-grid scale model. The approach relies on the dynamic procedure to reproduce the correct asymptotic behaviour of the turbulent flow near the wall, see e.g. Léziej and Mélais [11].

The second unclosed term, namely the sub-grid scale scalar flux \( J_{i}^{sgs} \), is modeled by an eddy-diffusivity approach, similarly to the sub-grid scale stresses. Here, a constant turbulent Schmidt-number \( \sigma_t \) relates the turbulent diffusion coefficient \( D_t \) to the turbulent viscosity \( \nu_t \).

\[ J_{i}^{sgs} \approx -D_t \frac{\partial \tilde{T}}{\partial x_i} \quad \text{with} \quad D_t = \frac{\nu_t}{\sigma_t} \quad \text{and} \quad \sigma_t \approx 0.4 - 0.7 \] (6)

While this approach is the simplest approach available, it has been found to give accurate agreement in the past [1, 5, 9]. This work will first concentrate only on an isothermal case.

### 3.2 Numerical Procedure

The previously presented governing equations are solved by the three dimensional finite-volume CFD code FASTEST-3D. This code uses geometry-flexible, block-structured, boundary fitted grids and allows therefore to represent complex geometries as presented in this contribution. A collocated and cell-centred arrangement of the variables is used on the grid.

The spatial discretizations are performed by second-order central schemes, especially designed for complex grids by Lehnhäuser and Schäfer [10], except for the convective transport in the scalar equation. Here, a flux limiter scheme with total variation diminishing (TVD) properties has been used to ensure bounded and non-oscillatory transport as solutions for the mixture fraction [20]. The pressure-velocity coupling is achieved by a SIMPLE similar procedure. A semi-implicit Crank-Nicolson method of second order accuracy is employed for the time integration of the transport equations. Finally, the resulting set of linear equations is solved iteratively utilising Stone's strongly implicit procedure (SIP).

The code is parallelized by domain decomposition using the MPI message passing library. For further information on the details of the employed method refer to Durst and Schäfer [3].

<table>
<thead>
<tr>
<th>No.</th>
<th>grid</th>
<th>( S/S_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(a) coarse, (b) fine</td>
<td>1.00</td>
</tr>
<tr>
<td>(2)</td>
<td>coarse</td>
<td>0.50</td>
</tr>
<tr>
<td>(3)</td>
<td>coarse</td>
<td>0.75</td>
</tr>
<tr>
<td>(4)</td>
<td>coarse</td>
<td>2.00</td>
</tr>
</tbody>
</table>

\( w_b = 8.31 \text{ m/s}, D = 25 \text{ mm}, Re_D \approx 13350 \) and \( S_n = 0.55 \)

Table 1. Four cases investigated. Varying the swirl-number the properties of the flow shown in the last row of this table were kept constant.

### 4 NUMERICAL CONFIGURATION AND BOUNDARY CONDITIONS

For the numerical investigation different LES on two grids are performed as summarized in table 1. The first grid consists of a total number of \( 4.375 \cdot 10^5 \) points. Based on this coarse grid (a) the second grid (b) is a systematically refined grid which doubles the grid points in each direction so that a total number of \( 3.5 \cdot 10^6 \) grid points arises.

To capture the expected hydrodynamic instability and the recirculation zone the computational domain includes the region of the swirler nozzle without the inner and outer circumferential channels (see figure 2). Instead, corresponding inlet velocity profiles were prescribed on the lateral swirlier surfaces. The third inlet region consists of the annular slot. In contrast to the experimental investigation this inlet was fed with a dummy mass flow of \( 1 \text{ m}^3/\text{h} \) with no swirl component in order to analyze the mixing behavior. Here, the properties of air at ambient temperature and pressure are used.

The whole computational domain consists of the dual-swirler nozzle, the combustion chamber, the Laval-shaped outlet nozzle and the exhaust pipe. The radial direction in the exit plane of the swirler nozzle is discretized for the coarse grid with 70 cells and the circumferential direction with 68 cells, respectively. Altogether the numerical domain of the swirler nozzle consists of approximately 7.66 \( \cdot 10^7 \) grid points. The outlet nozzle is mapped with roughly \( 3.53 \cdot 10^4 \) and the exhaust pipe with \( 7.19 \cdot 10^4 \) cells.

### 5 COMPARISON OF RESULTS

To carry out a numerical experiment a validation is needed to trust the LES results. Therefore data of the two reference LES (1a) and (1b) and measurements are compared at several axial and radial positions depicted in figure 3. Additionally a time series is captured at the points T1 with the coordinates \( r = 0 \text{ mm}, x = 0 \text{ mm} \) and T2 with at \( r = 12 \text{ mm}, x = 6 \text{ mm} \). The coloring in figure 3 corresponds to the mean axial velocity of calculation (1a). As consequence of the swirl a strong recirc-
calculation zone is forming from inside the swirl-nozzle far into the combustion chamber. Furthermore a lateral recirculation zone similar to the recirculation in a backward facing step flow can be observed in the combustion chamber.

Figure 4 shows radial profiles of the time averaged axial velocity and the standard deviation of the axial velocity. Compared with experimental data the coarse as well as the fine LES capture most of the main features of the flow in the near nozzle region. Only the recirculation region is predicted too long. Both LES calculations tend to overestimate this feature of the flow.

Time averaged axial profiles of the axial and tangential velocity components are shown in figure 5 for the coarse LES and experiments. The axial quantities are in a very good agreement with experimental data. In a qualitatively manner the calculations predict the behaviour of the tangential flow correctly. Quantitatively these components are underestimated. So the perfect fit of the crosscorrelation \( (U'W')^{1/2} \) in the lower plots seems to be though coincidence.

Besides the evaluation of the one-point, one-time statistics presented in figures 3–5 the LES-method taps its full potential in predicting transient flow processes. To get information about coherent structures it is necessary to look at the energy spectra (equation 7) from the autocovariance of flow quantities in point \( x_i \). The normalized autocovariance (autocorrelation) for the quantity \( \varphi \) is given in equation 8.

\[
E_{\varphi\varphi}(\omega) = \frac{\langle \varphi' \varphi' \rangle}{\pi} \int_{-\infty}^{\infty} R_{\varphi\varphi,t} \cdot e^{-i\omega \Delta t} \, d(\Delta t)
\]  

(7)

\[
R_{\varphi\varphi,t}(x_i, \Delta t) = \frac{\langle \varphi'(x_i, t) \cdot \varphi'(x_i, t + \Delta t) \rangle}{\langle \varphi'^2(x_i, t) \rangle}
\]  

(8)

The autocorrelation function which was calculated from the experimental timeseries gives an evidence of the presence of a flow-dominating coherent structure. Applying the Fourier-analysis according to equation 7 the peak frequency of 398 Hz arises from experiments. Using the inner air-nozzle diameter \( D = 15.4 \, mm \) and the bulk velocity \( u_b = 8.31 \, m/s \) with the definition of the

\[
\langle \varphi' \varphi' \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} R_{\varphi\varphi,t} \cdot e^{-i\omega \Delta t} \, d(\Delta t)
\]  

(9)

Figure 3. Positions for the LES validation marked by lines in the cross-section. The colouring corresponds to the mean axial velocity \( \langle U \rangle \) of LES (1a).

Figure 4. Radial profiles of the time averaged axial velocity \( \langle U \rangle \) (left) and the standard deviation \( \langle U'^2 \rangle^{1/2} \) of the axial velocity (right) for LES (1a & 1b).

Figure 5. Axial profiles of the time averaged axial (tangential) velocity \( \langle U \rangle \) (\langle W \rangle), the standard deviation \( \langle U'^2 \rangle^{1/2} \) (\langle W'^2 \rangle^{1/2}) of the axial (tangential) velocity and the crosscorrelation \( \langle U'W' \rangle^{1/2} \) for LES (1a).
Strouhal-Number (equation 9) this frequency is equivalent to 
\[ Str \approx 0.74 \] which is within the range expected for swirl-flow instabilities.

\[
Str = \frac{f \cdot L}{U} = \frac{f \cdot D}{u_b} \tag{9}
\]

Figure 6 is a result of the described temporal analysis carried out for the LES (1a) and (1b). Compared to the experiment the frequencies \(~ 340 \, Hz\) for LES (1a) and \(~ 380 \, Hz\) for LES (1b) are too low. But still, the prediction especially for the calculation on the fine grid is very good.

The instability analysis of Michalke [14] in this case results in a frequency of approximately 304 Hz and a positive growth rate of the first azimuthal mode. Taking into account that the analytical velocity profiles used in the instability analysis provides only a rough mapping of the real flow field, this analysis gives a reasonable prediction of the instability behavior, and is qualitatively comparable to the results of the LES calculations.

Overall the validation of the reference LES (1a) and (1b) on two different grids shows that the one-point statistic as well as the estimation of the dominating frequency by means of Fourier-analysis deliver good qualitativ and quantitativ results. Only the tangential velocity component in the case of the coarse LES (1a) deviates strongly from the experimental findings. This suggests a strong influence of the grid refinement on this component which is in accordance to the estimation of the frequencies for the two LES.

6 FLOW ANALYSIS

As pointed out in the previous section the expected coherent structure could be detected by the simulations. Because of the dominating behaviour of this flow instability containing a big amount of the total flow energy the spatial structure and the mechanism of formation is described in this section. Therefore a vortex definition proposed by Jeong and Hussain [8], namely the \( \lambda_2 \)-criterion, is used to visualize the coherent structures in the flowfield. Regions of negative \( \lambda_2 \) which is defined as the second largest eigenvalue of the tensor (10) capture the details of the structure.

\[
S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj} \tag{10}
\]

This method was developed especially for wall-bounded flows with strong shear. Hence it is appropriate for a close view on the vortical structures. Figure 7 originated from the LES (1a) clarifies the forming-mechanism of the rotating flow instability. Inside the inner swirler nozzle the fluid rotates like a rigid body around the principal axis. Reaching the boundary where recirculating fluid arises and which is identifiable by high values of the velocity gradient \( \partial u/\partial r \) the fluid bends out due to shear-stresses. These shear-stresses force the fluid on a helical line. Because of the impressed swirl this helical structure is characterized by two rotations. The superior motion is a precessing rotation of the whole helical structure around the principal axis and the inferior motion is a rotation around the helical line by itself. In the center of the helical structure a second branch which continues the rigid body rotation around the principal axis can be observed.

![Figure 6](image6.png)

**Figure 6.** Power density spectrum \( E_{vv} \) (right) and normalized autocorrelation \( R_{vv} \) (left) at \( T'1 \) obtained from LES case (1a) (bottom) and LES case (1b) (top).

![Figure 7](image7.png)

**Figure 7.** Isosurfaces of the instantaneous \( \lambda_2 \) taken from the LES (1a). The colouring corresponds to positive values of the velocity gradient \( \partial u/\partial r \). The flow instability is identifiable by the helical structure.
In the case (1a) the obtained precession frequency was calculated from the LES-data to $\sim 340 \text{ Hz}$ ($\text{Str} = 0.63$). Varying the swirl-number in the range $0.5 < S/S_n < 2.0$ according to the cases (2), (3) and (4) in table 1 results in a nearly linear Strouhal-Swirl-Number dependency, figure 8. The existence of the above described coherent helical structure could be shown for the cases (1a), (3) and (4). For case (2) this structure could not be observed. Figure 9 demonstrates this circumstance based on power spectra. The appearance of the adequate higher-order harmonic oscillations in the cases (3) and (4) is remarkable because this behaviour cannot be observed in the cases (1a) and (1b) with a swirl-number which is lying in between. Due to the width and the absolute value of the peak within the energy spectrum of case (2) a weakening of the coherent motion is associated with a swirl-number reduction.

Above $S/S_n \geq 0.75$ the main feature of each spectrum is a sharp peak which exceeds the remaining wave-number regions about one or two orders of magnitude. The reason for this distinctive flow dominating structure is due to the presence of a backflow-mechanism inside the swirler-nozzle for these cases. In fact a flow instability arises in case (2) but there is no strengthening backflow. Hence only a minor influence of the rotating instability on the flow can be observed. The formation of instabilities in the near nozzle region is a result of the swirling flow with or without backflow. In a qualitativ manner the present investigation confirms the analytical results of Michalke [14].

As displayed in figure 10 two flow features of the investigated configuration are produced by narrowing the cross sectional area using a Laval-shaped nozzle. These consist, first, of an acceleration of the fluid due to the conservation of mass and angular momentum, and, second, of the penetration of the coherent structure through this nozzle. Figure 10 visualizes the development of the vortex field inside the combustion chamber and the penetration mechanism by means of the $\lambda_2$-criterion. As shown above, the formation of the helical structure is driven by the velocity gradient $\partial u/\partial r$. In the streamwise direction this gradient decreases, so that the structure fizzles out. In figure 10 the helical structure is covered by circular vortices originated from the outer recirculation zone which can be well identified in figure 3.

Only the vortex which branched off from the helical structure can be encountered in the rear of the combustion chamber. There the fluid is accelerating towards the Laval nozzle. Be-
cause of the yielding negative velocity gradient $\partial u/\partial r$ this vortical structure is forced on a circular path around the principal axis and is centred at the smallest cross sectional area inside the Laval nozzle.

Analyzing the timeseries in point $x = 133 \text{ mm}$, $r = 0 \text{ mm}$ results in energy spectra are plotted in figure 11. Considering these spectra on the coarse grid it is noticeable that peaks of high energy arise at about $1200 \text{ Hz}$ for the cases (1a), (2) and (3). The peak in the spectrum of case (4) is very weak at $425 \text{ Hz}$. However these are not as sharp as the peaks corresponding to the flow instability above the swirler nozzle. In contrast to the energy distribution in point $x = 0 \text{ mm}$, $r = 0 \text{ mm}$ it is possible to detect a peak value in the energy spectrum of the axial velocity $E_{uu}$ beside the peaks in $E_{vv}$ and $E_{ww}$. The peak frequency of $\sim 1200 \text{ Hz}$ is about 3 to 4 times bigger than the first modes shown in figure 9. This is approximately the same factor as the geometric ratio of $D/D_{\text{min}}$. The Strouhal-Swirl-Number dependency at this point is illustrated by figure 8 (right).

Within the swirl-number range $0.5 < S/S_n < 1.0$ a nearly constant Strouhal-number of $Str = 2.3$ is obtained. At $S/S_n = 2.0$ the swirl-number drops to $Str = 0.45$.

This phenomena is explainable with a strong recirculation into the Laval nozzle occurring at the high swirl-number and which does not appear at $0.5 < S/S_n < 1.0$. The recirculation zone is reaching streamup nearly up to the throat and blocks the penetration of the coherent structure.

To get an impression of the structure characteristics which appear, figure 12 shows integral lengthscales $l_{\phi\phi}$ at several axial positions for the cases (1a), (2) and (4). For the calculation of the lengthscale, the spatial autocorrelation is required, which is defined in equation 11.

\[
R_{\phi\phi,x}(x_i, \Delta x_i) = \frac{\langle \phi'(x_i) \phi'(x_i + \Delta x_i) \rangle}{\langle \phi'^2(x_i) \rangle \langle \phi'^2(x_i + \Delta x_i) \rangle}^{1/2} \tag{11}
\]

The lengthscale is formally defined as the integral over the complete range of the spatial lag as it is shown in equation 12.

\[
l_{\phi\phi}(x_i) = \frac{1}{2} \int_{-\infty}^{+\infty} R_{\phi\phi,x}(x_i, \Delta x_i) \, d\Delta x_i \tag{12}
\]

Here the lengthscales are calculated in radial direction. The distribution of these lengthscales show the dissimilarity of the flow for different swirl numbers. But the order of magnitude is very similar for all cases.

A key feature in order to simulate non-premixed combustion is the mixture of the fuel jet with the swirling air streams. This mixture is described via the mixture fraction approach, as described in section 3. The radial profiles of the mixture fraction are given in figure 13. One can see the good mixing properties of a swirl burner, as used in this configuration, since already at $x = 7 \text{ mm}$, the mixture is almost homogeneous. The fuel jet mixes immediately in the shear layer between the two streams.
7 TURBULENT NOISE GENERATION

One of the main intentions of this paper is to investigate the sensitivity effects of some operating conditions, such as swirl number, on the noise production. It was shown in section 6 that the breakdown of vortices into substructures is influenced by these conditions. Last but not least the energy peaks are dedicated to diverse oscillating modes. Following Tang and Ko [18] it can likely be expected that the observed breakdown and interaction processes are responsible for different noise generating mechanisms. To show this, we follow Ewert and Schröder [4] who considered vortex sound problems and proposed a set of acoustic perturbation equations (APE) which is appropriate for the use in combination with incompressible flow simulations.

They found that the fluctuation of the Lamb vector (equation 13)

$$L_i = \varepsilon_{ijk} \omega_j u_k \rightarrow L'_i = (\varepsilon_{ijk} \omega_j u_k)'$$

with $$\omega_j = \varepsilon_{ijk} u_i u_k$$ plays an important role for the generation of vortex sound. As one of their results, they identified $$L'_i$$ as one of the major source terms besides the non-linear and entropy terms. To gain a statistical view on the distribution of vortex noise sources in the combustor, we therefore evaluate in the following the major source terms besides the non-linear and entropy terms.

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8 CONCLUSIONS

Based on an extensive temporal and spatial analysis of the flow, the importance and the size of the flow structure have been pointed out within a wide range of the flowfield. The mechanism of inducing a secondary flow instability in the convergent-divergent outlet nozzle has been clarified, and vortex noise sources linked to the fluctuation of the Lamb vector have been evaluated. A direct dependence upon the operating conditions (swirl number) could be shown. The extension of these results to reacting case is left for future investigation.

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