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AERODYNAMICALLY STABILIZED NANOSATELLITE: PROBLEMS OF DESIGN

I.V. Belokonov, I.A. Timbai

Belokonov I.V. Professor, Dr. of Eng. Sci., Head of Space Researches Department of Samara State Aerospace University, Russia, E-mail: ibelokonov@mail.ru

Timbai I.A. Professor, Dr. of Eng. Sci., Samara State Aerospace University, Russia, E-mail: timbai@mail.ru

Contents

1. Introduction

2. The selecting of the design parameters providing single-axis stabilization

3. The averaged ballistic coefficient

4. Analysis of the possibility of three-axle stabilization

5. SamSat-QB50 and SamSat-218d as examples of applied utilization of obtaining results

6. Conclusions

1. Introduction

The most of nanosatellites are launched on low-Earth orbits.
It is advisable for attitude stabilization use the aerodynamic forces.

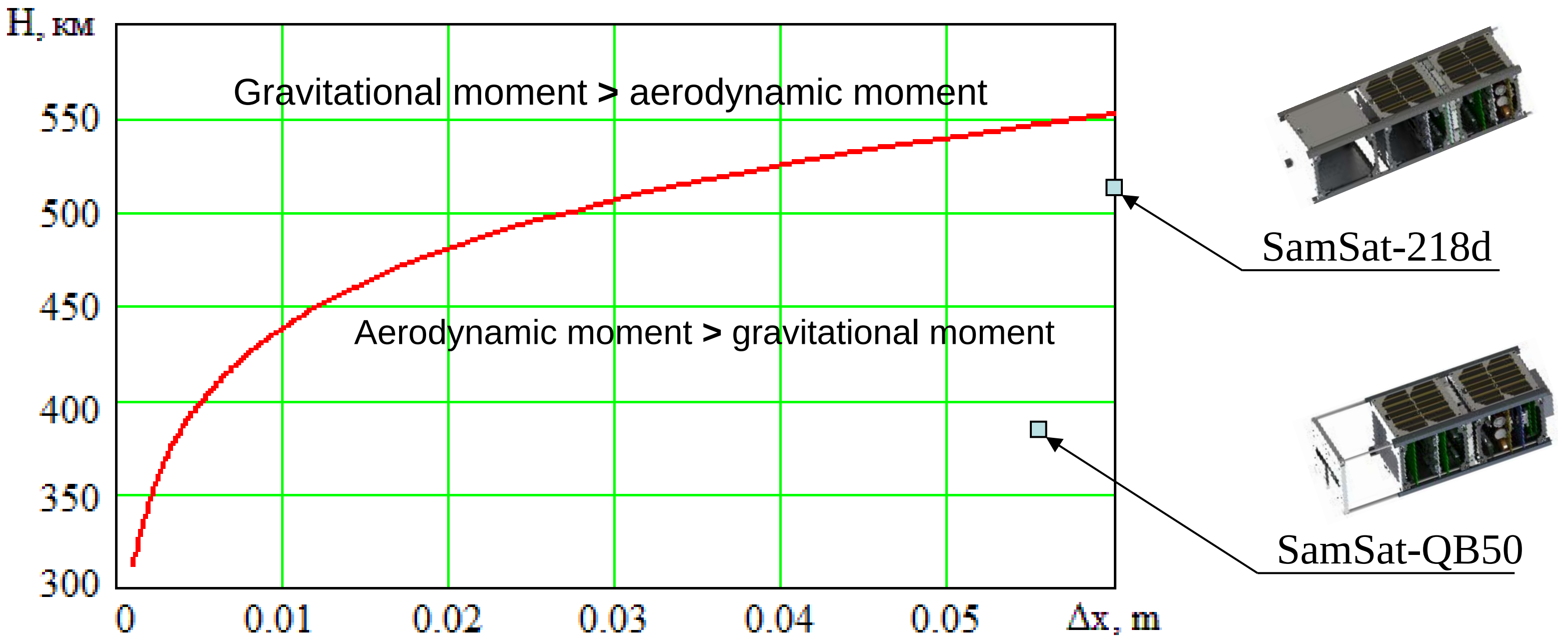


Figure 1. The ranges of orbit altitude and static stability factor, where aerodynamic moment exceeds gravitational moment for CubeSat 3U standard

The problem of ensuring of the aerodynamic stabilization of CubeSat nanosatellite in known scientific papers was considered in a deterministic statement.

In this paper the problem of selecting the design parameters (statical stability factor, length, longitudinal moment of inertia) of the aerodynamically stabilized nanosatellite is considered in a probabilistic formulation.

Nanosatellite planar angular motion

Approximate model of angular motion in the plane of the circular orbit with respect to the trajectory reference frame

$$\ddot{\alpha} - a(H)\sin\alpha - c(H)\sin 2\alpha = 0\,, \qquad (1)$$

where

α is the angle of attack; H is the flight altitude;

$a(H) = a_0 Slq(H)/J_n$ is coefficient associated with aerodynamic restoring moment;

$c(H) = 3(J_n - J_x)(\omega(H))^2/(2J_n)$ is coefficient associated with the gravitational moment;

$q(H) = V^2\rho(H)/2$ is velocity head; V is flight speed;

$\rho(H)$ is atmospheric density; $\omega(H) = \sqrt{\mu/(R_E + H)^3}$ is the angular orbital velocity of the nanosatellite;

R_E is radius of the Earth; μ is Earth's gravitational parameter;

J_x is the inertia longitudinal moment; $J_n = J_y = J_z$ is the inertia transverse moment.

Energy integral of system (1) for H=const

$$\dot{\alpha}^2/2 + a\cos\alpha + c\cos^2\alpha = E \qquad (2)$$

Nanosatellite spatial motion

Change of the angle of attack nanosatellite can be described by the equation:

$$\ddot{\alpha} + (G - R\cos\alpha)(R - G\cos\alpha)/\sin^3\alpha - a\sin\alpha = 0\,, \qquad (3)$$

$$R = J_x\omega_x / J_n = \text{const}\,, \qquad G = R\cos\alpha + (-\omega_y\cos\varphi_n + \omega_z\sin\varphi_n)\sin\alpha = \text{const}\,,$$

where *R* and *G* are projections of a kinetic moment vector on the longitudinal axis of the nanosatellite and on the mass center velocity vector, normalized with respect to the transversal moment of inertia,

φ_n is the angular velocity of proper rotation,

ω_x, ω_y, ω_z are components of the angular velocity in the body-fixed frame of reference

Integral of energy of system (3) for constant *H* :

$$\dot{\alpha}^2/2 + (R^2 + G^2 - 2RG\cos\alpha)/(2\sin^2\alpha) + a\cos\alpha = E\,. \qquad (4)$$

Maximum value of the angle of attack is determined by the equation:

$$(R^2 + G^2 - 2RG\cos\alpha_{\max})/(2\sin^2\alpha_{\max}) + a\cos\alpha_{\max} - E = 0\,. \qquad (5)$$

2. The selecting of the design parameters providing single-axis stabilization

Cumulative distribution function of the maximum angle of attack

If the value of the initial transverse angular velocity $\dot{\alpha}_0$ is distributed according to the Rayleigh law: $f(\dot{\alpha}_0) = \frac{\dot{\alpha}_0}{\sigma^2} \exp\left(-\frac{\dot{\alpha}_0^2}{2\sigma^2}\right)$

The cumulative distribution function:

$$F(\alpha_{\max}) = 1 - \exp\left(\frac{-a(\cos\alpha_{\max} - \cos\alpha_0) - c(\cos^2\alpha_{\max} - \cos^2\alpha_0)}{\sigma^2}\right) \qquad (6)$$

If the value $\dot{\alpha}_0$ is distributed according to the uniform law: $f(\dot{\alpha}_0) = \begin{cases} \frac{1}{\dot{\alpha}_{0\max}}, & \dot{\alpha}_0 \in [0, \dot{\alpha}_{0\max}] \\ 0, & \dot{\alpha}_0 \notin [0, \dot{\alpha}_{0\max}] \end{cases}$

The cumulative distribution function :

$$F(\alpha_{\max}) = \frac{\sqrt{2a(\cos\alpha_{\max} - \cos\alpha_0) + 2c(\cos^2\alpha_{\max} - \cos^2\alpha_0)}}{\dot{\alpha}_{0\max}} \qquad (7)$$

where a is coefficient associated with aerodynamic restoring moment;

c is coefficient associated with the gravitational moment;

I.V. Belokonov, A.V Kramlikh, I.A. Timbai, Low-orbital transformable nanosatellite: research of the dynamics and possibilities of navigational and communication problems solving for passive aerodynamic stabilization, Proceedings of 2th IAA Conference on Dynamics and Control of Space System, Roma, Italy, 24-26 march 2014, IAA-AAS-DyCoSS2-14-04-10.

Formulas for the selection of design parameters of aerodynamically stabilized nanosatellite standard CubeSat

If the value $\dot{\alpha}_0$ is distributed according to the Rayleigh law:

$$d = \frac{\Delta x}{J_n} lb \geq d_r = \frac{\pi \sigma^2 \ln(1 - p^*)}{4 c_0 \left(\cos \alpha^*_{\max} - \cos \alpha_0 \right) q(H)} \qquad (8)$$

If the value $\dot{\alpha}_0$ is distributed according to the uniform law:

$$d = \frac{\Delta x}{J_n} lb \geq d_r = \frac{\pi \left(\dot{\alpha}_{0\max} p^* \right)^2}{8 c_0 \left(\cos \alpha_0 - \cos \alpha^*_{\max} \right) q(H)} \qquad (9)$$

where Δx is the static stability factor (the distance measured from the center of mass to the nanosatellite (NS) geometric center), l is the NS length, b is the NS width, α_0 is the initial value of spatial angle of attack (the angle between the longitudinal axis and velocity vector), $J_n = J_y = J_z$ is the inertia transverse moment, $q(H) = V^2 \rho(H)/2$ is the velocity head, V is the flight speed, H is the orbit altitude, $\rho(H)$ is the atmospheric density, $c_0 = 2.2$ is the drag force coefficient.

I.V. Belokonov, I.A. Timbai, Selection of design parameters of aerodynamically stabilized nanosatellite standard CubeSat, Scientific and Technological Experiments on Automated Space Vehicles and Small Satellites, Abstracts of the Third International Conference, 2014, Samara, Russia, p. 103-106

Selection of design parameters of aerodynamically stabilized nanosatellite CubeSat 3U

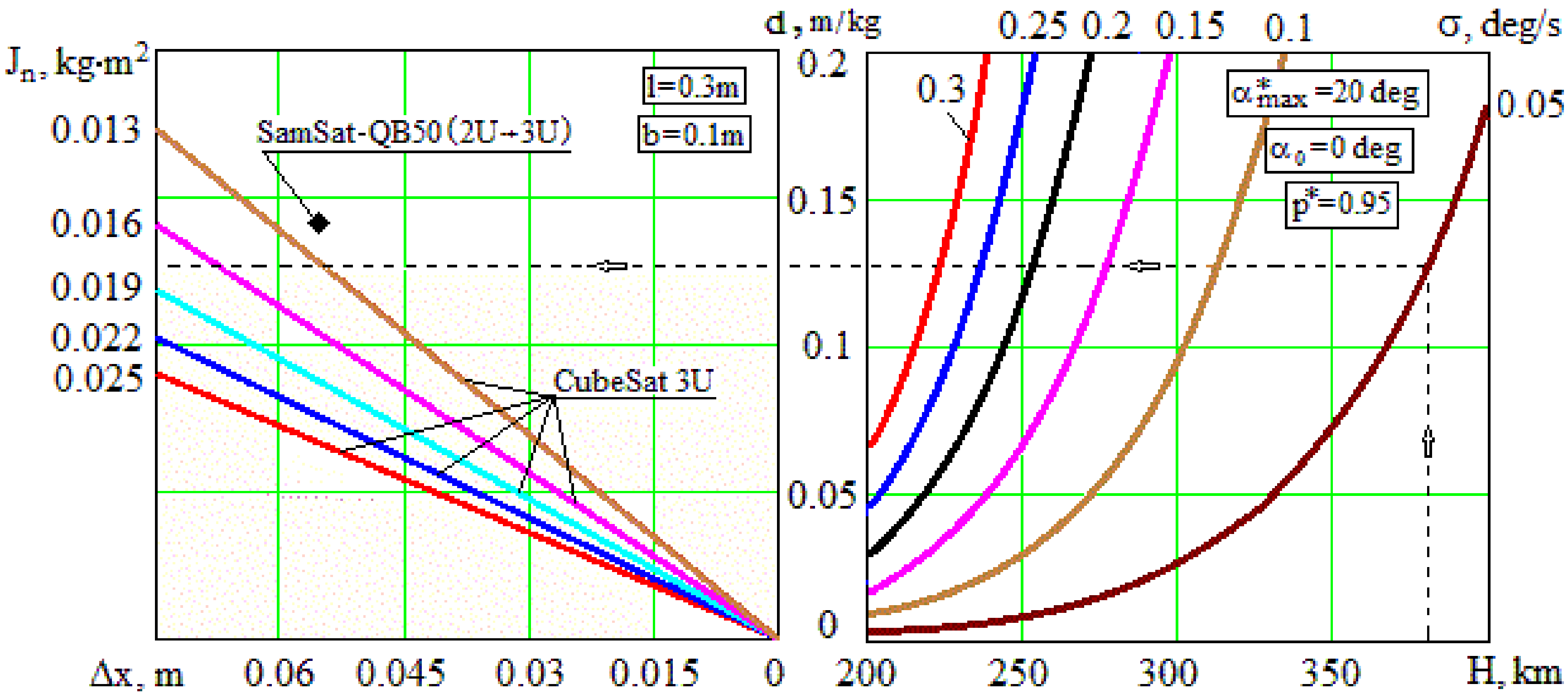


Figure 2. Nomogram to select structural parameter of nanosatellite depending on the altitude *H* and the parameter values σ at $\alpha^*_{\max} = 20$ deg , $p^* = 0.95$, $\alpha_0 = 0$.

3. The averaged ballistic coefficient

The ballistic coefficient:

$$\beta = \sum_{i=0}^{n} a_{\sigma_i} \cos^i \alpha \qquad (10)$$

where a_{σ_i} are approximation coefficients of ballistic coefficient by trigonometric series in powers $\cos\alpha$, n is the number of harmonics.

The averaged on oscillation period of the angle of attack of the ballistic coefficient:

$$\bar{\beta} = \sum_{i=0}^{n} a_{\sigma_i} N_i \qquad (11)$$

where

$$N_i = \sum_{r=0}^{i} C_i^r h_{2(i-r)} \quad (i = 0,1,2,...), \qquad C_i^r = i!x_1^r/[(i-r)!r!], \qquad h_2 = 2\eta(E - k_1^2 K)/K, \qquad k = \sqrt{A/2\eta},$$

$$h_{2j} = 2\eta[(2j-2)(2k^2-1)h_{2j-2} + (2j-3)k_1^2 A h_{2j-4}]/(2j-1) \qquad (j = 2, 3, 4,...), \qquad h_0 = 1, \qquad k_1 = \sqrt{1-k^2},$$

$$\eta = \sqrt{1 - 2(a_1 x_1 - b_1)/(1-x_1^2) + [(a_1 - b\ x_1)/(1-x_1^2)]^2}, \qquad a_1 = (R^2 + G^2)/(-4a), \qquad b_1 = RG/(-2a),$$

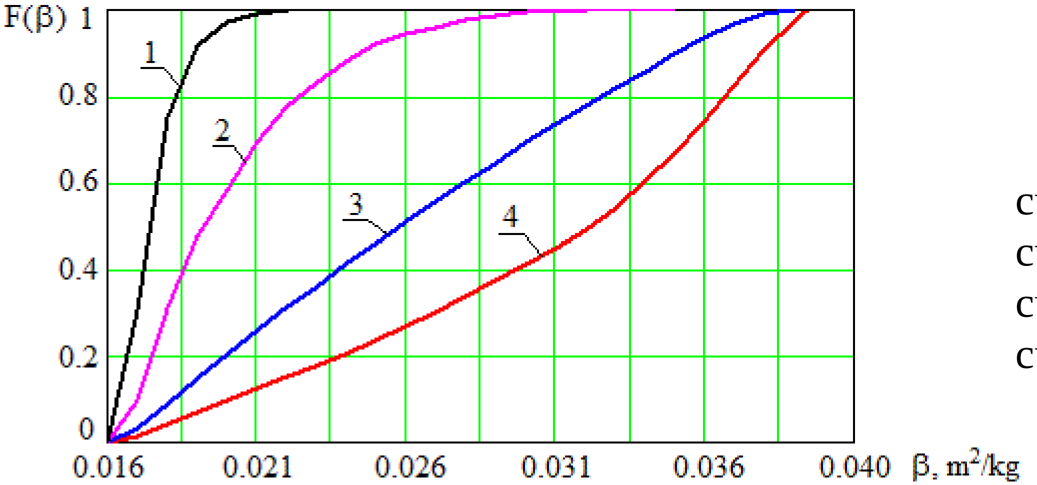
$$A = x_2 - x_1, \qquad x_1 = \cos(\alpha_{\max}), \qquad x_2 = \cos(\alpha_{\min}) = \eta - (a_1 - b_1 x_1)/(1-x_1^2), \qquad a = a_0 qSl/J_n,$$

$K(k)$, $E(k)$ are the complete elliptic integrals of the first and second kinds.

The maximum angle of attack:

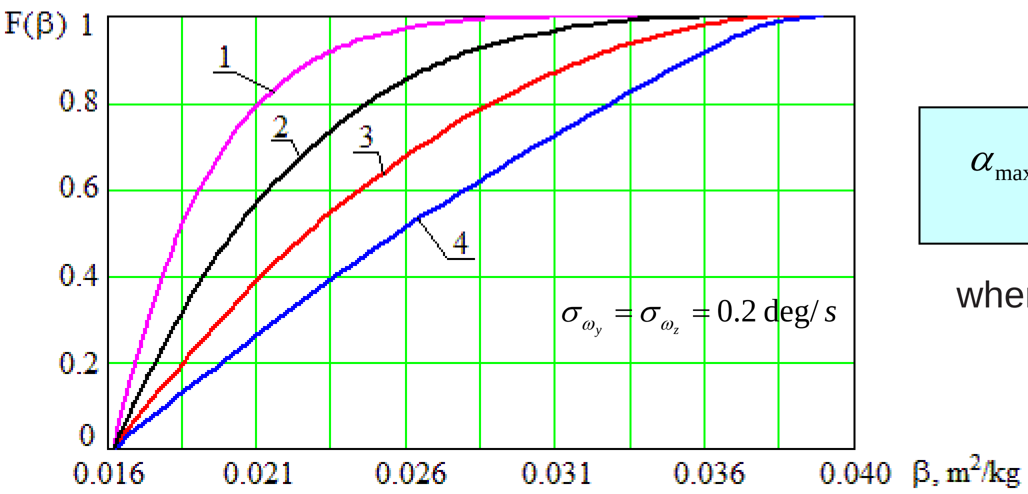
$$(R^2 + G^2 - 2RG\cos\alpha_{\max})/(2\sin^2\alpha_{\max}) + a\cos\alpha_{\max} - E = 0 \qquad (12)$$

The averaged ballistic coefficient



curve 1 - $\sigma_{\omega_y} = \sigma_{\omega_z} = 0.05$ deg/s, $\sigma_{\omega_x} = 0.017$ deg/s;
curve 2 - $\sigma_{\omega_y} = \sigma_{\omega_z} = 0.1$ deg/s, $\sigma_{\omega_x} = 0.033$ deg/s;
curve 3 - $\sigma_{\omega_y} = \sigma_{\omega_z} = 0.2$ deg/s, $\sigma_{\omega_x} = 0.067$ deg/s;
curve 4 - $\sigma_{\omega_y} = \sigma_{\omega_z} = 0.3$ deg/s, $\sigma_{\omega_x} = 0.1$ deg/s.

Figure 3. The cumulative distribution function of averaged ballistic coefficients of nanosatellite SamSat-QB50 (*H* = 380 km).



$$\alpha_{\max}(H) = 2\,\mathrm{asin}\sqrt{4D/\pi - 2D^2/\pi^2 - D^3/\pi^3 - 2.5D^4/\pi^4}, \quad (13)$$

where $D = D(H) = I_0/(8\sqrt{-a(H)})$,

$$I_0 = 8\sqrt{a(H_0)}(E - k_1^2 K).$$

Figure 4. The cumulative distribution function of averaged ballistic coefficients of nanosatellite SamSat-QB50: curve 1 – altitude *H* = 230 km, 2 - *H* = 280 km, 3 - *H* = 330 km, 4 - *H* = 380 km.

4. Analysis of the possibility of three-axle stabilization

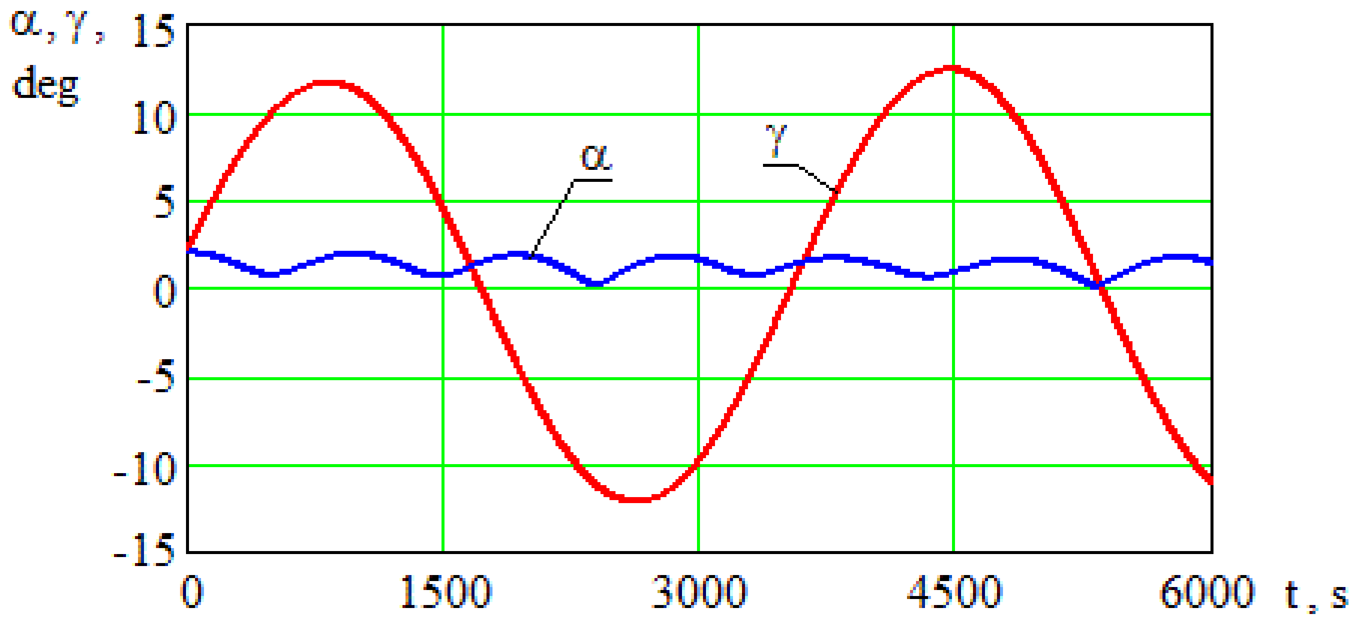
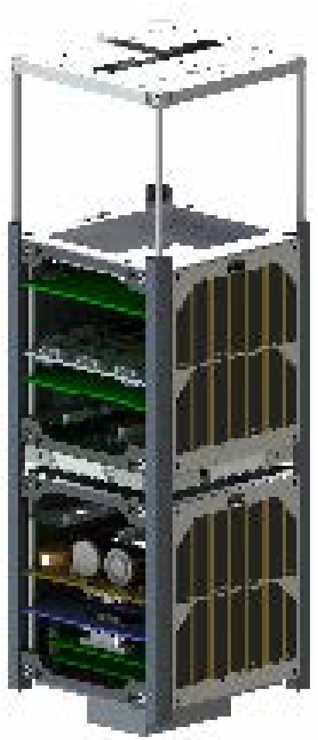


Figure 5. Changing the angular values of the time (*H* = 380 km).

α is the angle of attack,
γ is the angle of deviation of the transverse axis nanosatellite from the plane flight,

the ratio of the transverse moments of inertia $J_y/J_z = 1.2$,
the initial values of the components of the angular velocity:
ω_{x_0} =0.02 deg/s, ω_{y_0} =0.065 deg/s, ω_{z_0} =0.

5. SamSat-QB50 and SamSat-218d as an examples of applied utilization of obtaining results



J_n = 0.0083 kg·m² → J_n = 0.012 kg·m²,
Δx = 0.02 m → Δx = 0.055 m

Figure 6. SamSat-QB50 nanosatellite of the transformable design.

J_n = 0.014 kg·m²,
Δx = 0.06 m.

Figure 7. SamSat-218d nanosatellite.

Conclusions

Formulas for selecting the design parameters (statical stability factor, length, longitudinal moment of inertia) aerodynamically stabilized nanosatellite standard CubeSat which has the deviation the own longitudinal axis from velocity vector not more than acceptable with a given probability at the known errors of initial angular separation velocity are obtained.

The cumulative distribution function of the averaged ballistic coefficient is obtained.

The possibility of a three-axis stabilization of nanosatellite is shown.

The results reflect general conditions for the realization of the mission aerodynamically stabilized nanosatellite. The obtained results are used to create nanosatellite SamSat-QB50 as the part of the international project QB50.

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Thank you for your attention!