Sliding mode control for three-axis magnetic attitude

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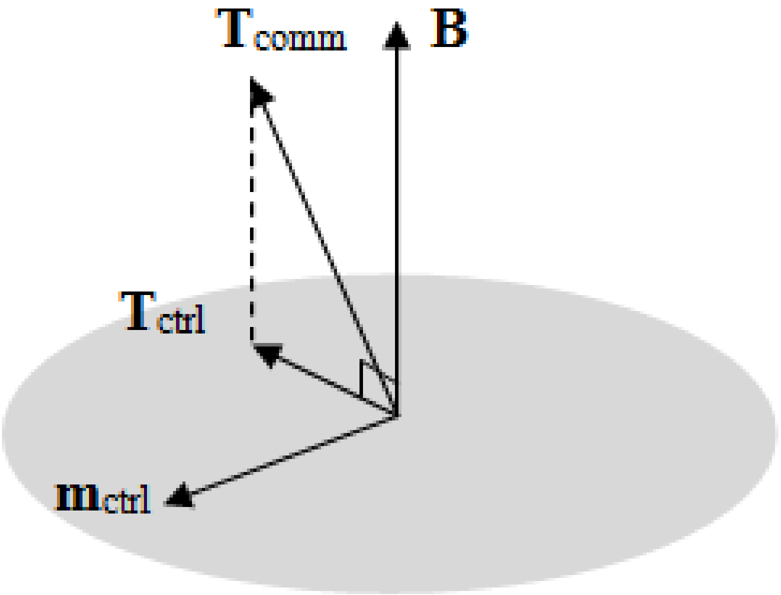
Underactuation

- Torque is perpendicular to geomagnetic field
- Conventional local control, i.e. PD controller, is unavailable
- Geomagnetic induction vector rotates with satellite orbital motion

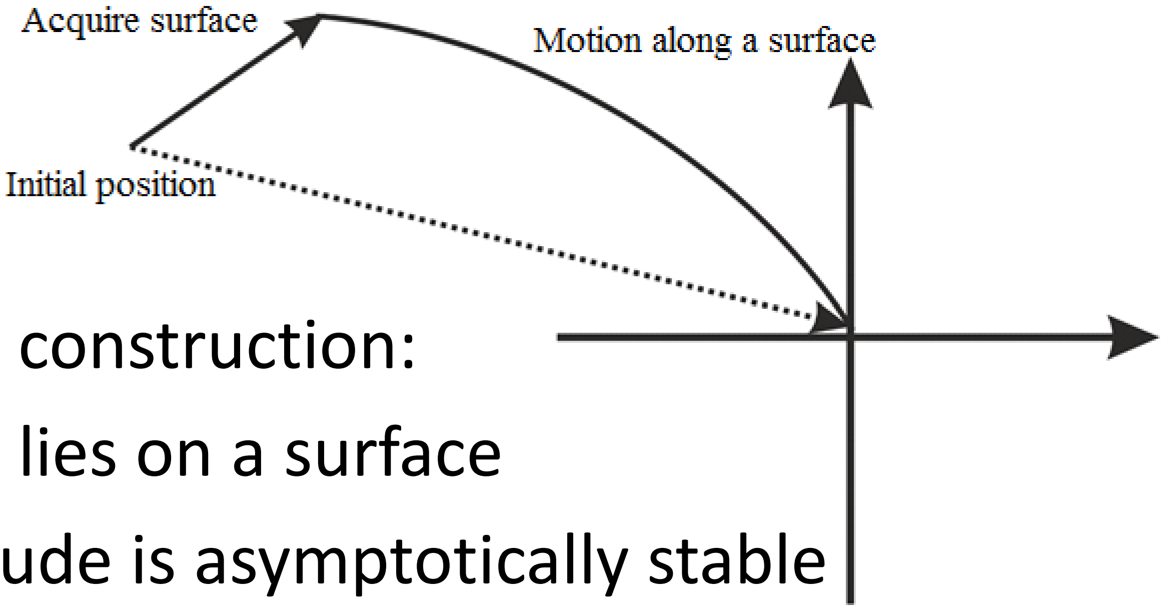
$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$



Accessible path

- Accessible trajectory may be constructed: for each time and position (orbital and angular) *necessary* torque is perpendicular to geomagnetic vector
- Geomagnetic vector rotation over one orbit allows every direction for the torque
- Ideally requires path prediction – solving boundary-value problem

Sliding control



Two stage control:

- Sliding surface construction:

  if phase vector lies on a surface

  necessary attitude is asymptotically stable
- Ensuring motion on the surface: control objective is changed, system order is reduced

Problem statement

- Rigid spacecraft
- Circular orbit
- Three ideal magnetorquers as actuators
- Current attitude is known
- Three axis attitude in inertial and orbital frames is necessary
- Gravitational disturbing torque is taken into account

Equations of motion

- Dynamical equations

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M} + \mathbf{M}_{ctrl} + \mathbf{M}_{dist}$$

- Sliding surface

$$\mathbf{x} = \lambda\left(\boldsymbol{\omega}, \mathbf{S}, t\right)\boldsymbol{\omega} + \boldsymbol{\Lambda}\left(\underline{\boldsymbol{\omega}, \mathbf{S}, t}\right)\mathbf{S} = 0,$$

$$\mathbf{S} = \left(a_{23} - a_{32},\ a_{31} - a_{13},\ a_{12} - a_{21}\right)$$

- Alternatively

$$\mathbf{x} = \lambda\boldsymbol{\omega} + 4q_0\boldsymbol{\Lambda}\mathbf{q} = 0$$

leads to

$$\frac{1}{2}\frac{d}{dt}\left(\mathbf{q}^T\mathbf{q}\right) = -4\lambda^{-1}q_0^2\mathbf{q}^T\boldsymbol{\Lambda}\mathbf{q} \le 0$$

Sliding surface construction

- Motion compliance with a sliding surface is ensured by

$$\dot{\mathbf{x}} = -\mathbf{J}^{-1}\mathbf{P}\mathbf{x}$$

- Surface parameters should allow

$$\dot{\lambda}\mathbf{J}\boldsymbol{\omega} + \lambda\mathbf{J}\dot{\boldsymbol{\omega}} + \mathbf{J}\dot{\boldsymbol{\Lambda}}\mathbf{S} + \mathbf{J}\boldsymbol{\Lambda}\dot{\mathbf{S}} = -\lambda\mathbf{P}\boldsymbol{\omega} - \mathbf{P}\boldsymbol{\Lambda}\mathbf{S}$$

- Control torque direction restriction necessitates particular surface parameters

Iterative method

- Position parameter derivative is approximated

$$\dot{\mathbf{\Lambda}} = \frac{\mathbf{\Lambda}(k+1) - \mathbf{\Lambda}(k)}{\Delta t}, \quad \lambda = const$$

- Surface parameters equation

$$\boxed{\mathbf{a} + \mathbf{\Lambda}(k+1)\mathbf{b} = \mathbf{m} \times \mathbf{d}}$$

where

$$\mathbf{a} = \left(-\dot{\lambda}\mathbf{J}\boldsymbol{\omega} + \lambda(\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} - \mathbf{M}) - \mathbf{\Lambda}(k)\left(\mathbf{J}\dot{\mathbf{S}} + \mathbf{P}\mathbf{S}\right) - \lambda\mathbf{P}\boldsymbol{\omega}\right)\Delta t + \mathbf{\Lambda}(k)\mathbf{J}\mathbf{S},$$

$$\mathbf{b} = -\mathbf{J}\mathbf{S}, \ \mathbf{d} = \lambda \Delta t \mathbf{B}$$

Iterative method

- New basis $\mathbf{e}_1 = \frac{\mathbf{d}}{|\mathbf{d}|}, \mathbf{e}_3 = \frac{\mathbf{d}\times\mathbf{b}}{|\mathbf{d}\times\mathbf{b}|}, \mathbf{e}_2 = \mathbf{e}_3\times\mathbf{e}_1$
- Surface parameters comply

$$\Lambda_{11}(k+1)b_1 + \Lambda_{12}(k+1)b_2 = -a_1$$

- Positional parameters:

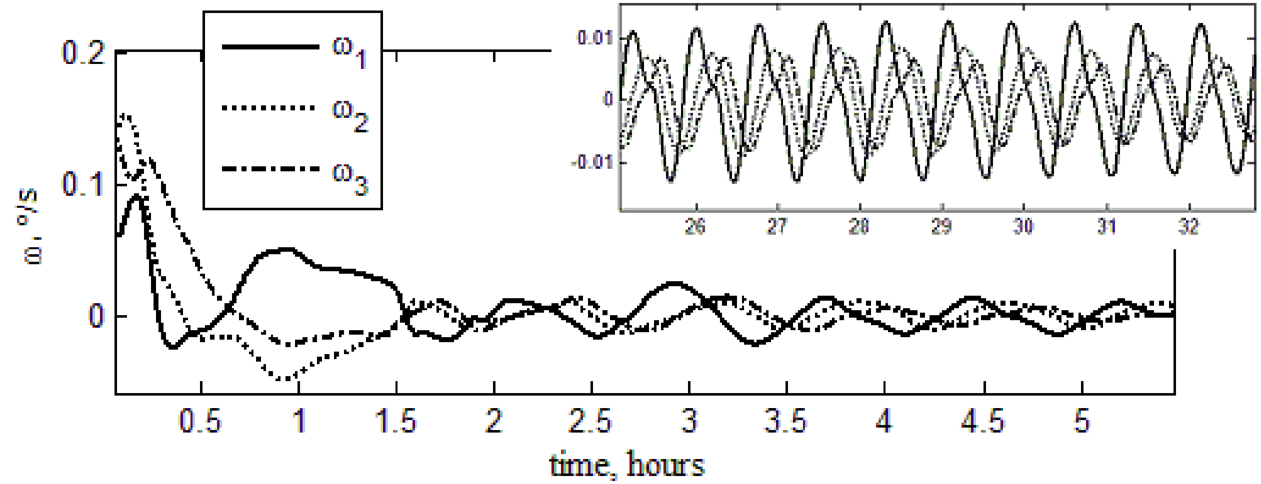
$$\Lambda_{11}(k+1) = \Lambda_{11}(k),$$

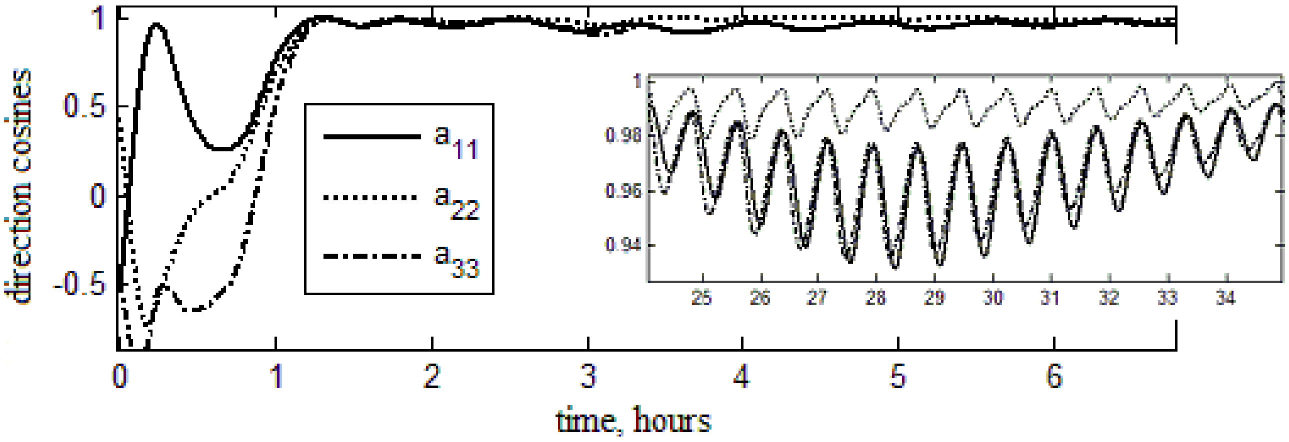
$$\Lambda_{12}(k+1) = \Lambda_{21}(k+1) = -\frac{a_1 + \Lambda_{11}(k+1)b_1}{b_2 + \delta b_2},$$

$$\Lambda_{22}(k+1) = \Lambda_0 + \frac{\Lambda_{12}^2(k+1)}{\Lambda_{11}(k+1)}$$

Modeling

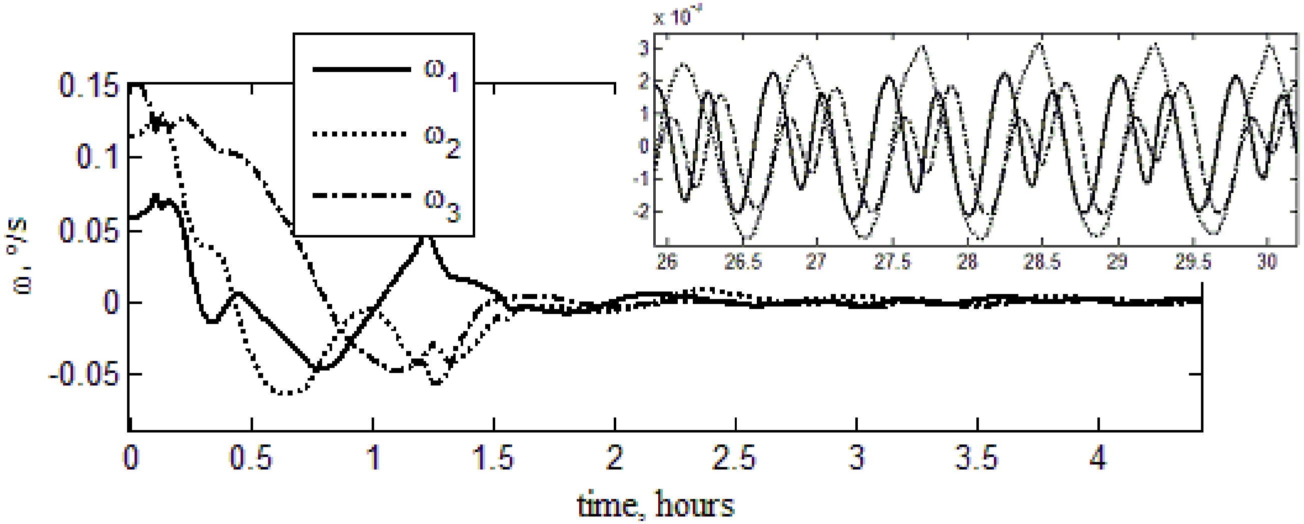
microsatellite, low orbit, inertial attitude – accuracy 15-20°

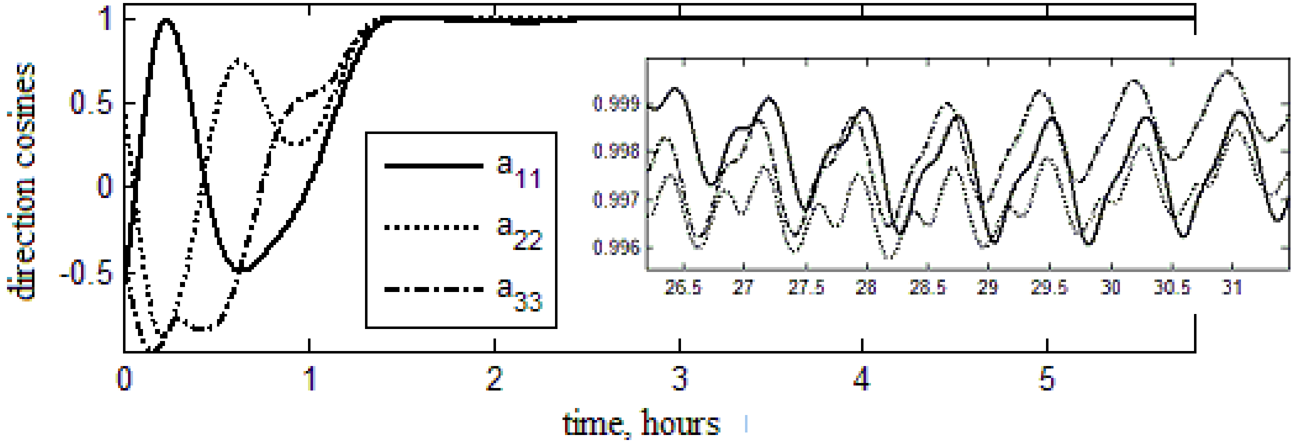




Modeling

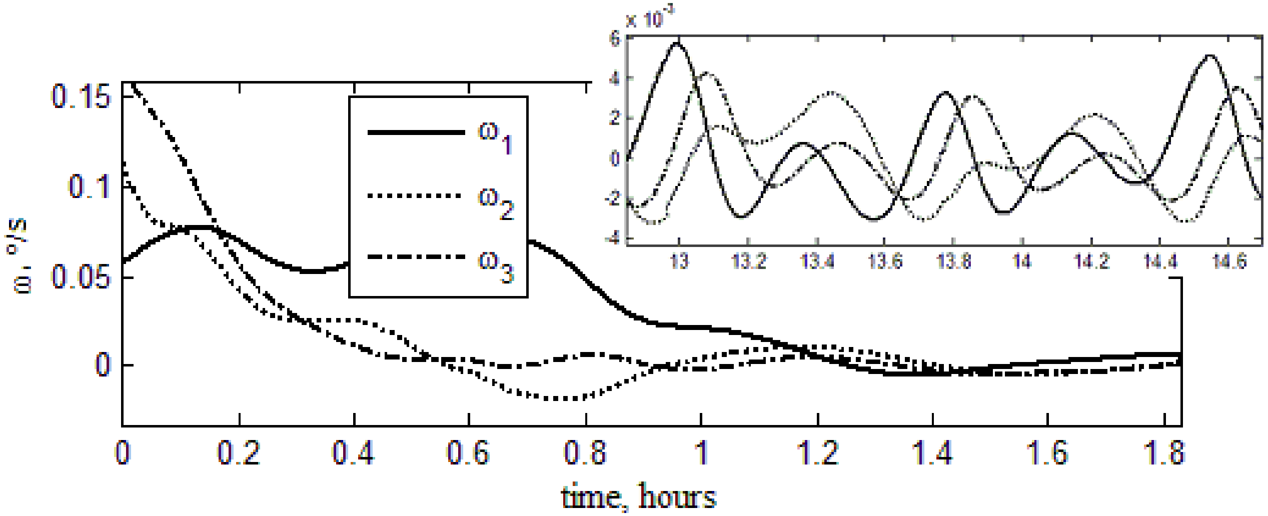
nanosatellite, low orbit, inertial space – accuracy 5°

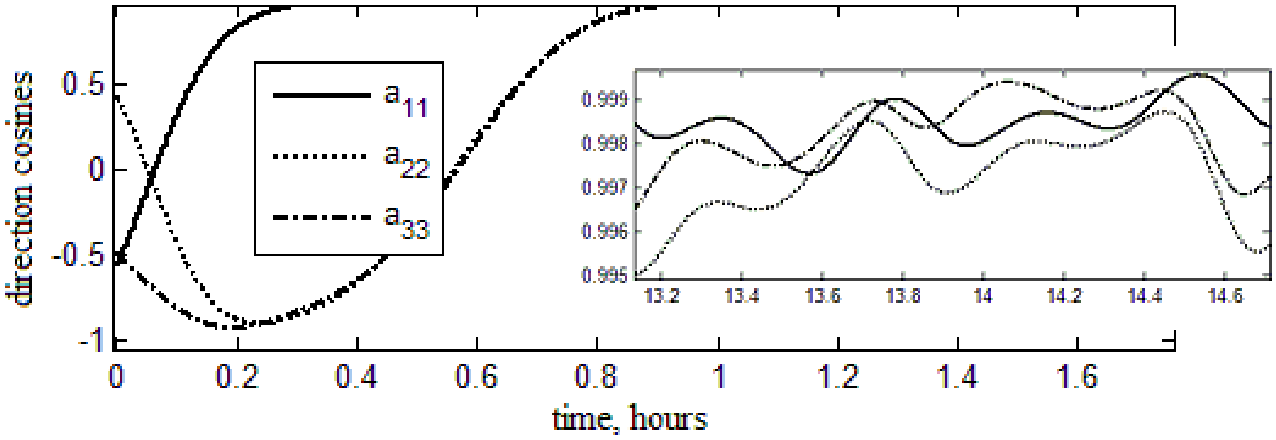




Modeling

CubeSat, low orbit, inertial/orbital – better than degree





Results

- Sliding control is used for magnetically-actuated satellite
- Accessible angular trajectory is found using varying sliding surface
- Iterative approach is devised for sliding surface parameters acquisition
- Accuracy is about few degrees for nanosatellites