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Attitude estimation, control and momentum dumping: a case study for CONASAT

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Topics

- Introduction
- Equations of motion
  - Control strategy
- Momentum dumping
- Attitude estimation
  - Reduced order covariance matrix
- Results
- Conclusions

Introduction

- CONASAT mission
- Replace the Brazilian satellites SCD-1 and SCD-2

- Collect environmental data from remote platforms and retransmit to the mission center
  - Rain volume, temperature, humidity, air pollution, ocean streams, environmental hazards

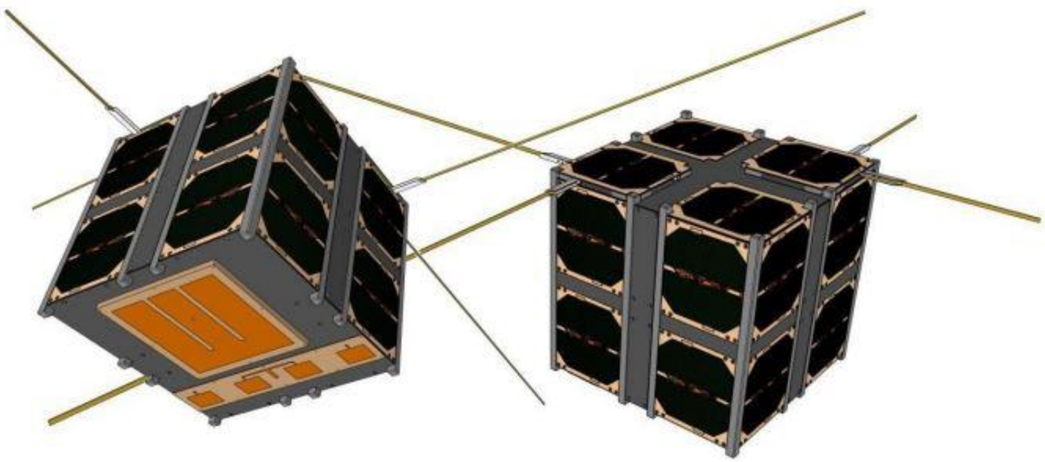
- Payload antenna – attitude pointed accuracy of 5°

Introduction

- 8U CubeSat
  - 2U for all subsystems
  - 2U – cold redundancy
  - 4U – extra surface for solar panels

- Sensors:
  - Solar sensors
  - Magnetic sensors
  - MEMS gyros

- Actuators:
  - Reaction wheels
  - magnetorquers



Objectives

- Estimate attitude and gyroscopic bias
  - Attitude must be represented in quaternions
- Implement a PID control strategy
- Test momentum dumping technics
  - CCPL – Conventional Cross Product Law
  - Bang-bang

Equations of motion

- Quaternion representation

- Have the following advantages:
  - Appropriate to computer implementation
  - Does not have singularities problems – as opposed to Euler angles
  - Less computational effort – no trigonometric functions

- The downside is that it does not have a direct physical interpretation

Equations of motion

- It is composed of:

$$\mathbf{Q} = \begin{pmatrix} \boldsymbol{\varepsilon} & \eta \end{pmatrix} \qquad \boldsymbol{\varepsilon} = \mathbf{a}\,\mathrm{sen}\,\frac{\theta}{2} \qquad \eta = \frac{\cos\theta}{2}$$

- And the rotation matrix representing the attitude:

$$\mathbf{C}_{ba} = \begin{pmatrix} \eta^2 + \varepsilon_1^2 - \varepsilon_2^2 - \varepsilon_3^2 & 2\left(\varepsilon_1\,\varepsilon_2 + \eta\,\varepsilon_3\right) & 2\left(\varepsilon_1\,\varepsilon_3 - \eta\,\varepsilon_2\right) \\ 2\left(\varepsilon_1\,\varepsilon_2 - \eta\,\varepsilon_3\right) & \eta^2 - \varepsilon_1^2 + \varepsilon_2^2 - \varepsilon_3^2 & 2\left(\varepsilon_2\,\varepsilon_3 + \eta\,\varepsilon_1\right) \\ 2\left(\varepsilon_1\,\varepsilon_3 + \eta\,\varepsilon_2\right) & 2\left(\varepsilon_2\,\varepsilon_3 - \eta\,\varepsilon_1\right) & \eta^2 - \varepsilon_1^2 - \varepsilon_2^2 + \varepsilon_3^2 \end{pmatrix}$$

Equations of motion

- The kinematic equation:

$$\dot{\mathbf{q}} = \frac{1}{2}\boldsymbol{\Omega}\,\mathbf{q} \qquad \boldsymbol{\Omega} = \begin{pmatrix} -\boldsymbol{\omega}^{\times} & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^{T} & 0 \end{pmatrix}$$

- Dynamic equation:

$$\mathbf{I}\dot{\boldsymbol{\omega}} = (\mathbf{I}\boldsymbol{\omega} + \mathbf{h}_r) \times \boldsymbol{\omega} + \mathbf{T}_{ext} - \mathbf{T}_r$$

- PID Control:

$$\mathbf{u}_k = -k_p\,\boldsymbol{\theta}_k - k_i \sum_{i=0}^{k} \boldsymbol{\theta}_i\,\Delta t - k_d\,\boldsymbol{\omega}_k$$

Momentum dumping

- CCPL – Conventional Cross Product Law

$$\mathbf{T} = -k\mathbf{\Delta h}$$

$$\mathbf{M} \times \mathbf{B} = -k\,\mathbf{\Delta h}$$

$$\mathbf{M} = g\,\mathbf{\Delta h} \times \mathbf{B}$$

- When the magnetic field is parallel to the reaction wheels speed, it is not possible to generate torque

Momentum dumping

- Bang-Bang strategy
- Takes the direction of the magnetic moment
- Applies the maximum momentum of the magnetorquer

$$\mathbf{M}_B = C\frac{(\Delta\mathbf{h}\times\mathbf{B})}{|\Delta\mathbf{h}\times\mathbf{B}|}$$

Attitude estimation

- The Kalman filter is a recursive method to optimally estimate the states of a system in the presence of noise
- For linear systems, have the following equations:

$$\mathbf{x}_{k+1} = \mathbf{\Phi}_{k+1,k}\,\mathbf{x}_k + \mathbf{\Gamma}_k\,\mathbf{w}_k$$

$$\mathbf{y}_k = \mathbf{H}_k\,\mathbf{x}_k + \mathbf{v}_k$$

Attitude estimation

- Prediction:

$$\bar{\mathbf{x}}_k = \mathbf{\Phi}_{k,k-1}\,\hat{\mathbf{x}}_{k-1} \qquad \bar{\mathbf{P}}_k = \mathbf{\Phi}_{k,k-1}\,\hat{\mathbf{P}}_{k-1}\,\mathbf{\Phi}_{k,k-1}^T + \mathbf{\Gamma}_k\,\mathbf{Q}_k\,\mathbf{\Gamma}_k^T$$

- Update:

$$\mathbf{K}_k = \bar{\mathbf{P}}_k\,\mathbf{H}_k^T\left(\mathbf{H}_k\,\bar{\mathbf{P}}_k\,\mathbf{H}_k^T + \mathbf{R}_k\right)^{-1}$$

$$\hat{\mathbf{P}}_k = \left(\mathbf{I} - \mathbf{K}_k\,\mathbf{H}_k\right)\bar{\mathbf{P}}_k$$

$$\hat{\mathbf{x}}_k = \bar{\mathbf{x}}_k + \mathbf{K}_k\left(\mathbf{y}_k - \mathbf{H}_k\,\bar{\mathbf{x}}_k\right)$$

Attitude estimation

- State equations – attitude and bias

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\boldsymbol{\Omega}\mathbf{q} \\ \mathbf{0} \end{bmatrix} + \mathbf{w}$$

$$\boldsymbol{\Phi}_{k+1,k} = \begin{pmatrix} \boldsymbol{\Phi}_{q|k+1,k} & \mathbf{0}_{4\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{I}_{3\times 3} \end{pmatrix}$$

$$\bar{\mathbf{q}}_{k+1} = \boldsymbol{\Phi}_{q|k+1,k}\,\hat{\mathbf{q}}_k$$

$$\bar{\mathbf{b}}_{k+1} = \hat{\mathbf{b}}_k$$

Gyro model:

$$\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} + \mathbf{b} + \boldsymbol{\eta}$$

$$\boldsymbol{\Phi}_{q|k+1,k} = \cos\left(\frac{|\boldsymbol{\omega}|\Delta t}{2}\right)\mathbf{I}_{4\times 4} + \frac{1}{|\boldsymbol{\omega}|}\,\mathrm{sen}\left(\frac{|\boldsymbol{\omega}|\Delta t}{2}\right)\boldsymbol{\Omega}(\boldsymbol{\omega})$$

Reduced order covariance matrix

- The quaternion covariance matrix presents a singularity in account of the unity module and extra element
- To avoid this problem, it is necessary to reduce the order of the covariance matrix:

$$\mathbf{P}^r = \mathbf{S}^T(\mathbf{q})\mathbf{P}\,\mathbf{S}(\mathbf{q}) \qquad \mathbf{Q}^r = \mathbf{S}^T(\mathbf{q})\mathbf{Q}\,\mathbf{S}(\mathbf{q})$$

$$\mathbf{S}(\mathbf{q}) = \begin{pmatrix} \boldsymbol{\Xi}(\mathbf{q}) & \mathbf{0}_{4\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{pmatrix} \qquad \boldsymbol{\Xi}(\mathbf{q}) = \begin{pmatrix} \eta & -\varepsilon_3 & \varepsilon_2 \\ \varepsilon_3 & \eta & -\varepsilon_1 \\ -\varepsilon_2 & \varepsilon_1 & \eta \\ \varepsilon_1 & -\varepsilon_2 & -\varepsilon_3 \end{pmatrix}$$

Reduced order covariance matrix

- Covariance matrix prediction

- Riccati equation for the reduced order filter:

$$\overline{\mathbf{P}}_{k+1}^{r} = \tilde{\mathbf{\Phi}}\,\overline{\mathbf{P}}_{k}^{r}\,\tilde{\mathbf{\Phi}}^{T} + \int_{t_k}^{t_{k+1}} \tilde{\mathbf{\Phi}}\,\mathbf{Q}^{r}\,\tilde{\mathbf{\Phi}}^{T}\,dt$$

- With:

$$\mathbf{\Lambda} = \mathbf{\Xi}^{T}\left(\overline{\mathbf{q}}_{k+1}\right)\mathbf{\Phi}_{q}\left(\hat{\boldsymbol{\omega}}_{k}\right)\mathbf{\Xi}\left(\hat{\mathbf{q}}_{k}\right)$$

$$\tilde{\mathbf{\Phi}} = \begin{pmatrix} \mathbf{\Lambda} & \mathbf{K}^{*} \\ \mathbf{0}_{3\times 3} & \mathbf{I}_{3\times 3} \end{pmatrix}_{6x6}$$

$$\mathbf{K}^{*} = -\frac{1}{2}\int_{t_k}^{t_{k+1}} \mathbf{\Lambda}\,dt$$

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Reduced order covariance matrix

- Covariance matrix update:

$$\hat{\mathbf{P}}_k^r = \left(\mathbf{I}_{6\times 6} - \tilde{\mathbf{K}}_k \tilde{\mathbf{H}}_k\right)\overline{\mathbf{P}}_k^r$$

$$\tilde{\mathbf{H}}_k = \mathbf{H}_k \mathbf{S}(\overline{\mathbf{q}}_k) \qquad \tilde{\mathbf{K}}_k = \overline{\mathbf{P}}_k^r \tilde{\mathbf{H}}_k^T \left(\tilde{\mathbf{H}}_k \overline{\mathbf{P}}_k^r \tilde{\mathbf{H}}_k^T + \mathbf{R}_k\right)^{-1}$$

- State update:

$$\hat{\mathbf{x}}_k = \overline{\mathbf{x}}_k + \mathbf{K}_k \left(\mathbf{y}_k - \mathbf{H}_k \overline{\mathbf{x}}_k\right)$$

$$\mathbf{K}_k = \mathbf{S}(\overline{\mathbf{q}}_k)\tilde{\mathbf{K}}_k$$

Results – Simulation parameters

- Simulation time: 18000s – equivalent to 3 orbits
- Circular orbit with 25° inclination and altitude of 630 km
- Initial attitude: 60°, 30° e 40° in Euler angles
- Angular velocity 0.6 rpm, 0.3 rpm e 0.9 rpm and null reaction wheels speed.
- Disturbance torque: residual magnetic moment of 0.01 Am^2.

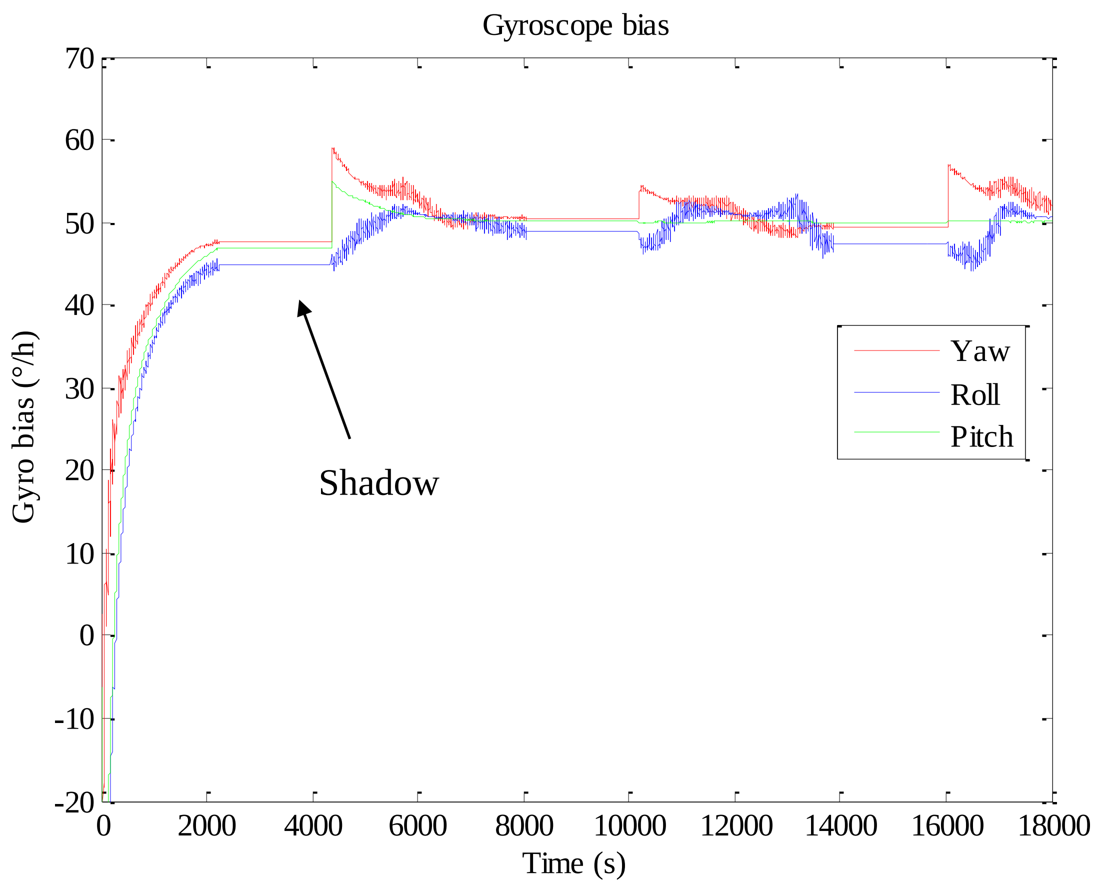
Results – Simulation parameters

- MEMS gyroscope bias and standard deviation: 50°/h and 5°/h, respectively
- Solar sensor and magnetic sensor noises: 0.5° and 1mG

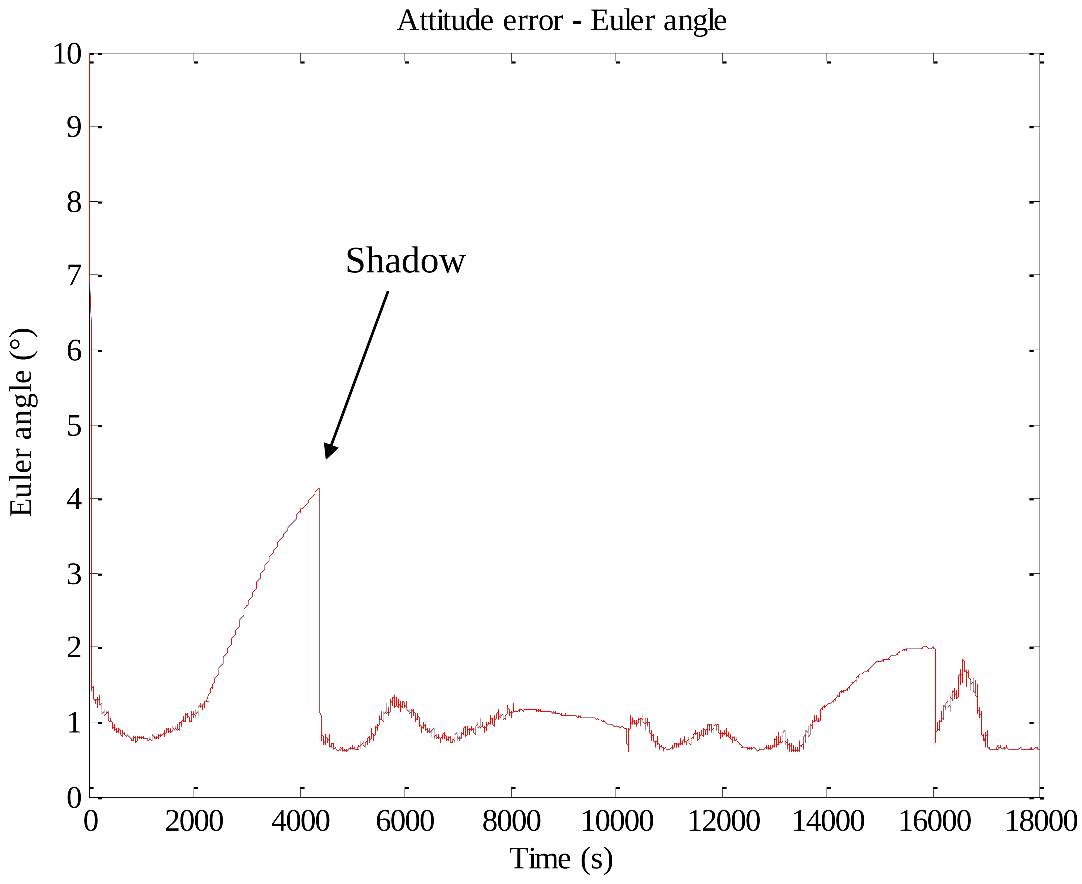
- Filter characteristics:

- Initial states $\mathbf{x}_0 = \mathbf{0}$
- Quaternions covariance: 0.0025 for the vector part and 0.000019 for the scalar
- Bias covariance: diagonal(0.000001)

Results

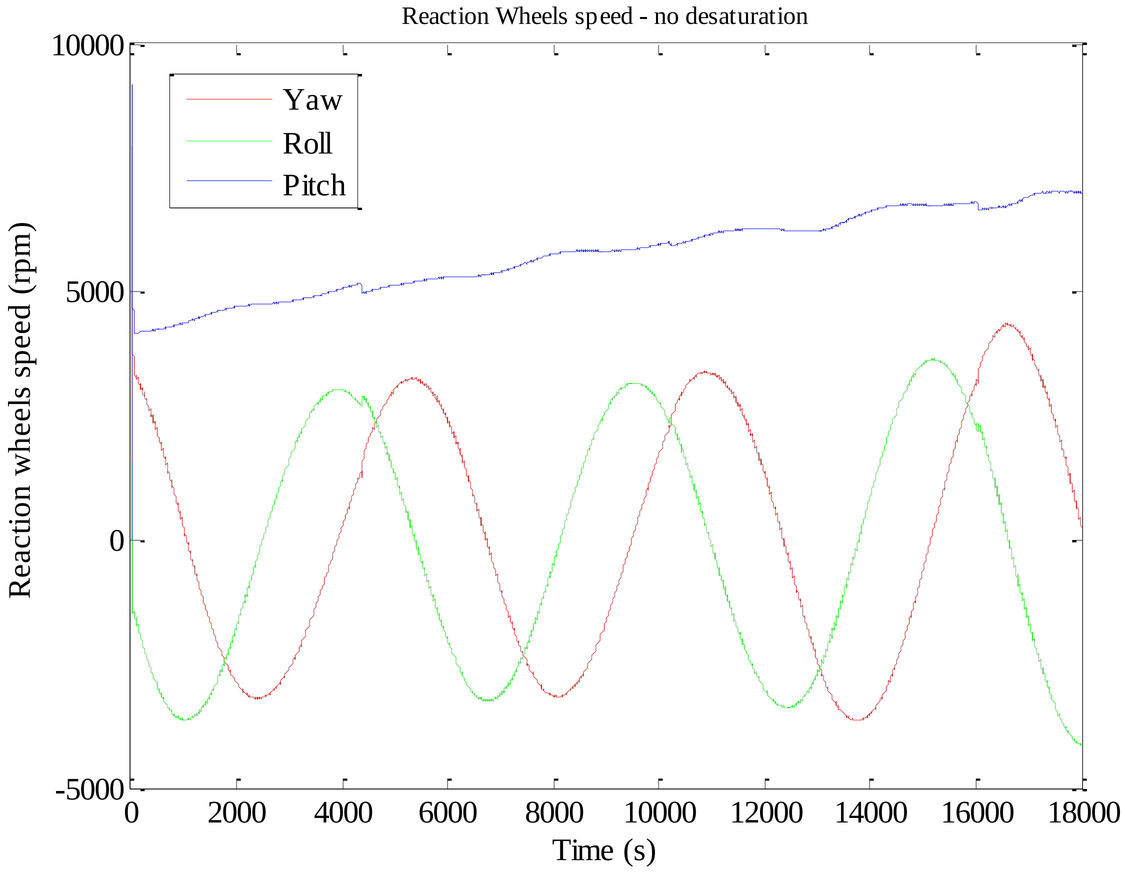

Gyroscope bias
Gyro bias (°/h)
Time (s)
70
60
50
40
30
20
10
0
-10
-20
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
Shadow
Yaw
Roll
Pitch

Results



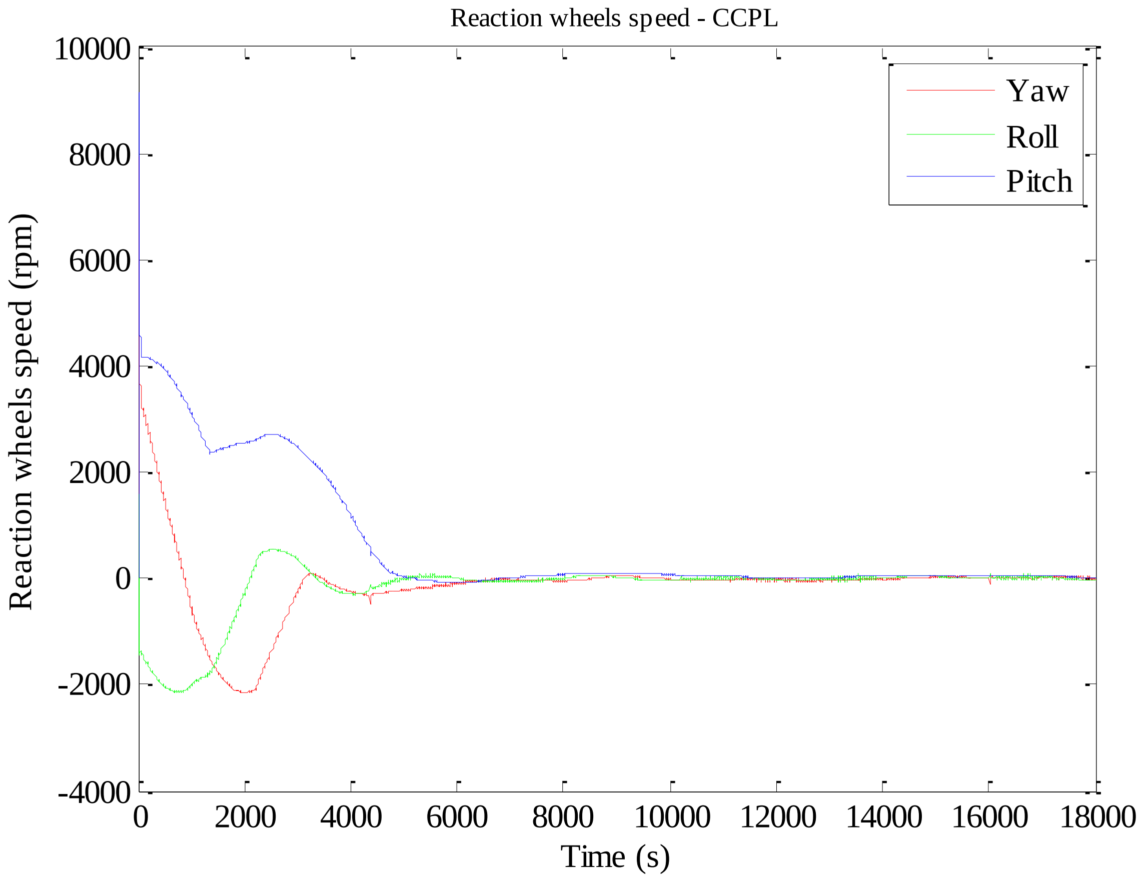
Attitude error - Euler angle
Shadow
Euler angle (°)
Time (s)
0
1
2
3
4
5
6
7
8
9
10
0
2000
4000
6000
8000
10000
12000
14000
16000
18000

Results



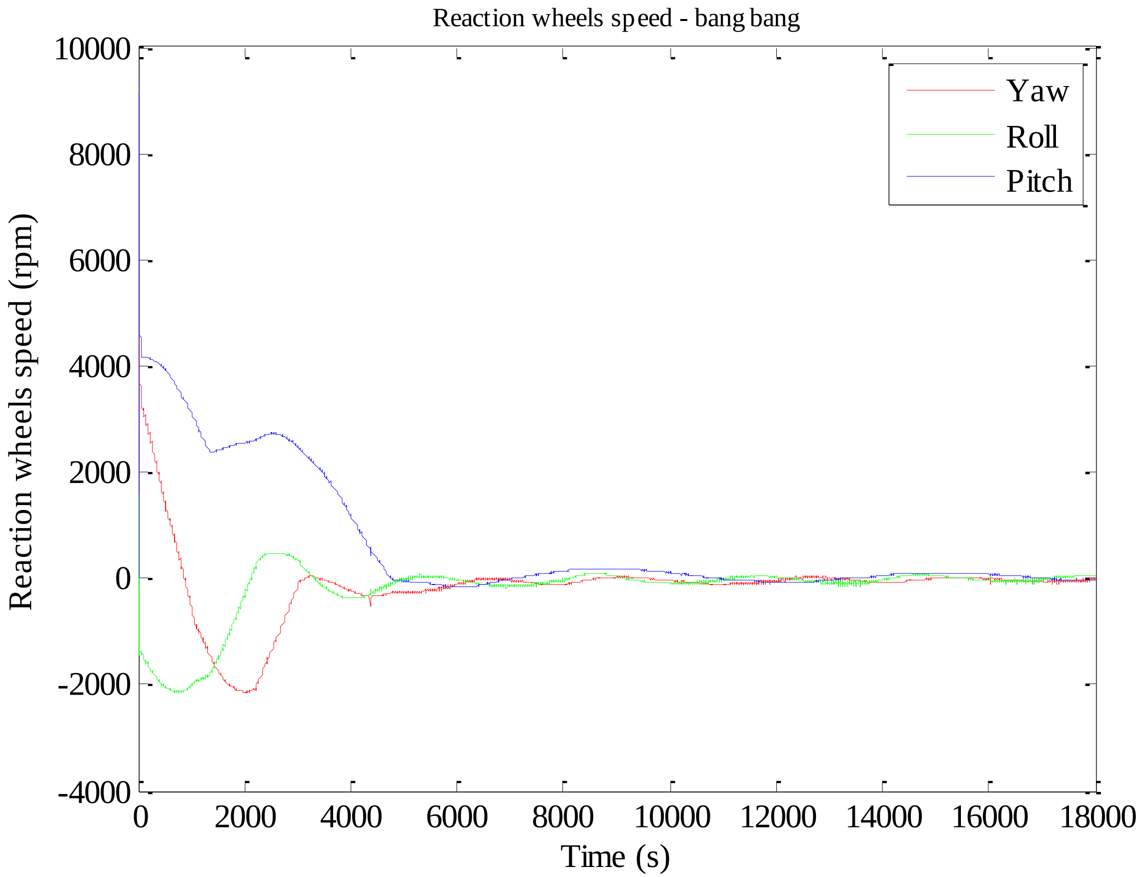
Reaction Wheels speed - no desaturation
Yaw
Roll
Pitch
Reaction wheels speed (rpm)
10000
5000
0
-5000
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
Time (s)

Results



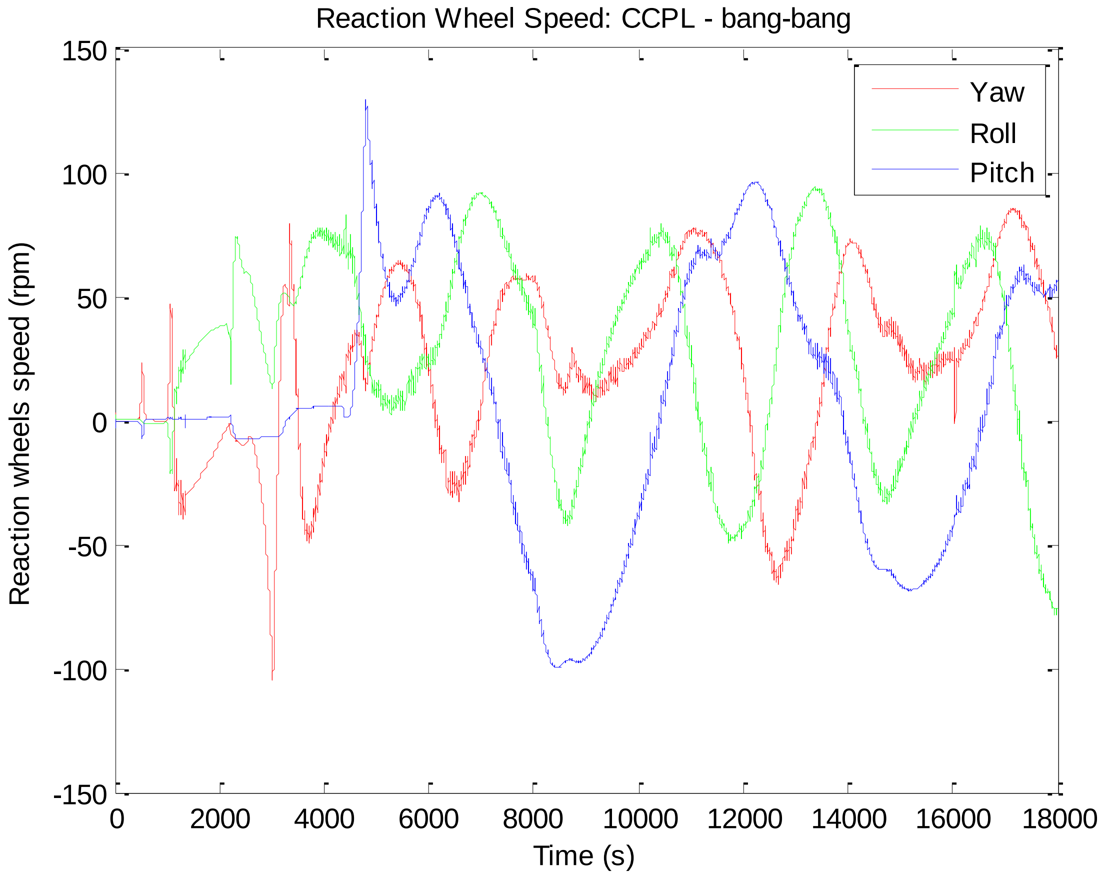
Reaction wheels speed - CCPL
Yaw
Roll
Pitch
Reaction wheels speed (rpm)
10000
8000
6000
4000
2000
0
-2000
-4000
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
Time (s)

Results

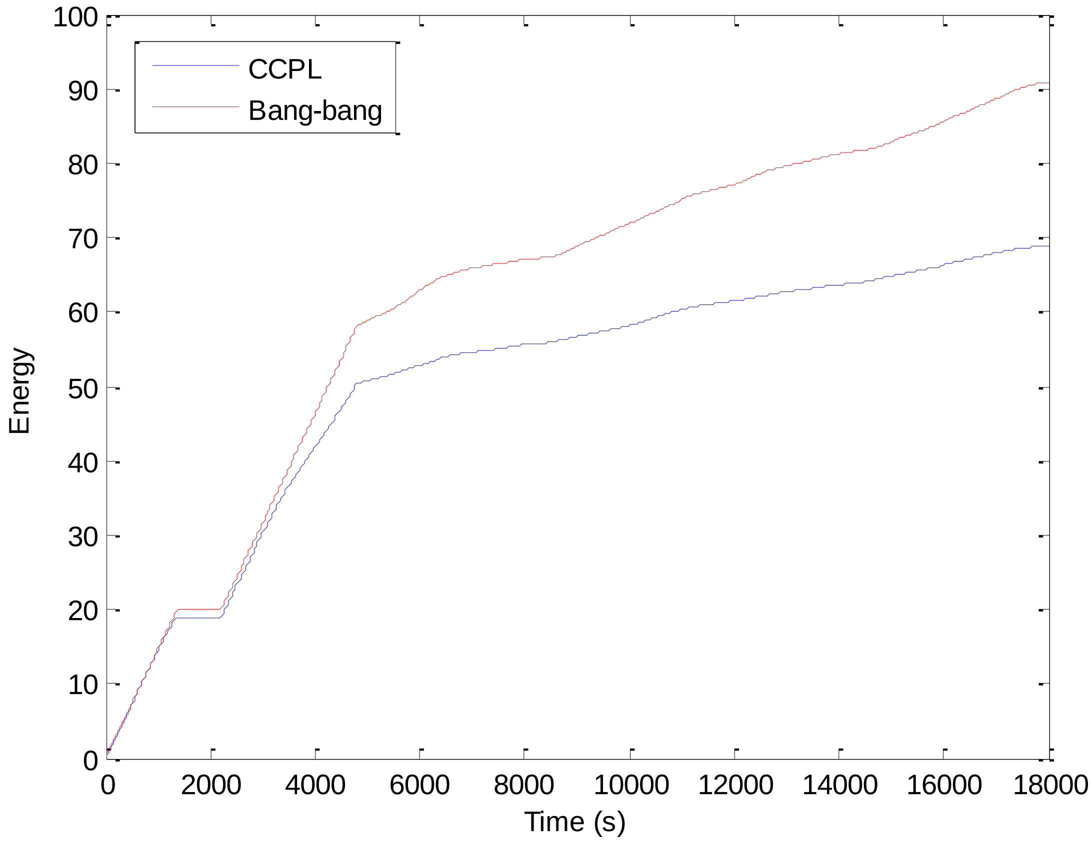


Reaction wheels speed - bang bang
10000
8000
6000
4000
2000
0
-2000
-4000
Reaction wheels speed (rpm)
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
Time (s)
Yaw
Roll
Pitch

Results


Reaction Wheel Speed: CCPL - bang-bang
Yaw
Roll
Pitch
Reaction wheels speed (rpm)
Time (s)
150
100
50
0
-50
-100
-150
0
2000
4000
6000
8000
10000
12000
14000
16000
18000

Results


CCPL
Bang-bang
Energy
Time (s)
0
10
20
30
40
50
60
70
80
90
100
0
2000
4000
6000
8000
10000
12000
14000
16000
18000

Conclusions

- CONASAT configuration
- Equations of motion - attitude represented by quaternions and the adopted PID control strategy.
- CCPL and bang-bang methods.
- Kalman filter - covariance matrix singularity, an inherent problem when using quaternions.

- Achieved < 5° attitude error and the desaturation of the reaction wheel’s

Thank you!

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