ABSTRACT

Telescopes with laser illumination are used for identification of characteristics of space debris and other subjects moving in space. The nanosatellite REFLECTOR equipped with a passive gravity-gradient attitude control system combined with magnetic hysteresis damper and a several sets of laser retroreflectors was launched for calibration of these telescopes on 10th of December, 2001. There are no other components at the satellite. It is tracked by ground-based stations combined in IRLS. Besides the orbital parameter detection and telescope calibration it is supposed to determine parameters of the satellite attitude motion by processing of laser orbital tracking data. Apart from a self-dependent interest it allows us to verify models of the magnetization with hysteresis and to specify methods of determination of attitude control system parameters. In the paper it is shown the way to obtain precession velocities and angle of nutation. First results of computation are presented. Then evolution of the velocities and angles allows us to make conclusion about the effectiveness of the hysteresis effect.

1. REFLECTOR SATELLITE

REFLECTOR is a satellite of 7 kg mass with telescopic boom, square basis and six retroreflectors, four are installed at the corners of the basis and two are at the boom. Also, 18 magnetic hysteresis rods are installed in three perpendicular plates. General view and drawing of the REFLECTOR are shown in Fig.1.

2. PROBLEM DESCRIPTION

While moving satellite is lighted up with laser beam and time of return signal movement is measured. Through this time, the distance from a retroreflector to the target plane which is perpendicular to the laser beam and crosses the satellite at its center of mass is calculated. During tracking session we obtain a lot of such
measurements. A samples of the measurements are represented in Fig.2. We obtain characteristics of attitude motion using an assumption about regular precession of the satellite.

3. MODEL DESCRIPTION

We assume that the satellite is axially symmetric and that during a tracking session of 10-15 min. length it is weakly affected by external torques. Thus, its motion represents a regular precession with the angular velocity $\Omega$, angular velocity of spin rotation $\omega$ and nutation angle $\theta$.

Let us determine at what curves points representing measurements described above lie. We introduce two Cartesian reference frames. The first frame is OXYZ with OZ axis directed along axis of precession, others axes OX and OY are arbitrary but perpendicular to OZ axis and O to be at the center of mass of the satellite. The second frame is the body-fixed frame Oxyz.

The transformation matrix from the frame OXYZ to the frame Oxyz is as following

$$
\begin{pmatrix}
\cos \varphi \cdot \cos \theta \cdot \cos \psi - \sin \varphi \cdot \sin \psi \\
\cos \varphi \cdot \cos \theta \cdot \sin \psi + \sin \varphi \cdot \cos \psi \\
\cos \varphi \cdot \sin \theta
\end{pmatrix}
$$

where $\theta$ is constant, $\varphi = \Omega t$, $\psi = \omega t$. To obtain distance from a retroreflector to the target plane we transpose radius-vector of the retroreflector to the OXYZ frame and multiply it scalarwise by normal to the target plane. As a result, we obtain a linear combination of sine functions with different frequencies. At the same time functions corresponding to the reflectors installed at the boom contain the single sine harmonic since there is only one nonzero component of radius-vector of these reflectors. Cyclic frequency of this sinusoid is equal to $\Omega$. Analyzing spectrum of others functions we can
obtain $\omega$. Knowing boom length and amplitude of movements of the reflector installed at it we obtain $\theta$.

**Fig.2.** Samples of preprocessed result of tracking sessions

4. TECHNIQUE DESCRIPTION

Results of measurements are preprocessed manually to distinguish points of one curve among plenty of points at the picture. We took into consideration only pictures which didn’t give cause for ambiguity. Then, the curve is interpolated at $2^n$ uniformly distributed points to enable the fast Fourier transformation. Spectrum of this curve is approximated by Nyquist-Kotelnikov formula

$$F(\omega) = \sum_{i=-k}^{k} F_i \sin \pi (i\Delta - \omega) \over \pi (i\Delta - \omega)$$

where $k=2^{n-1}$, $F_i$ is sampled value, $\Delta$ is sampling interval and maximum corresponding to required harmonic is selected.

5. THEORETICAL ESTIMATIONS

In [1] the formula is given

$$I \frac{d\omega}{dt} = V_{rod} \left( \int HdB \right)_{aver} \over 2\pi$$

where $I$ is moment of inertia of the satellite, $V_{rod}$ is total volume of hysteresis damper and $\left( \int HdB \right)_{aver}$ is the average square of the hysteresis loop. Substitute the following quantities of parameters: $I=1.76$ kg⋅m$^2$, $\left( \int HdB \right)_{aver} \sim 7$ T⋅A/m. A number of hysteresis rods affected with REFLECTOR rotation is 12. Length of every rod is 110 mm, and diameter is 0.3 mm. So, $V_{rod} \sim 10^{-7}$ m$^3$ then $\frac{d\omega}{dt} \sim 7 \cdot 10^8$ s$^{-2} \sim 5 \cdot 10^3$ s$^{-1}$/day and $df/dt = (2\pi)^{-1} \frac{d\omega}{dt} \sim 10^3$ s$^{-1}$/day. That is, the frequency of precession decreases with decrement $10^{-3}$ s$^{-1}$/day. Let us compare this evolution with results obtained by processing of measurements.
6. RESULTS

The graph with results of proceeded tracking sessions is shown in Fig.3. Each point corresponds to specific tracking session, throughout axis of abscissa is date of session, zero corresponds to 01.01.2002, throughout axis of ordinate is calculated frequency of precession. Approximation by least square method gives us a straight line described by formula $f\sim -3 \cdot 10^{-4}t + 1.7 \cdot 10^{-2}$ where $t$ is measured in day unit.

![Fig.3](image)

Fig.3. Calculated by data of single tracking session frequencies

7. CONCLUSION

We can see that even such a rough approximation gives us rather close to theoretical estimations values. At a later time it is planned to specify model, to introduce in it disturbing factors and to develop a method for obtaining more characteristics of the attitude motion. From the other hand, it is planned to proceed the same data with Kalman filter method.

8. ACKNOWLEDGEMENT

This work was partially supported by Russian Basic Research Foundation, Russian Academy of Sciences (Project ? 89 of 6th Competition of Young Scientists) and DAAD (Leonard Euler Scholarship, Referat ? 325).