ABSTRACT

This paper presents an original tracking algorithm designed for star trackers adopting CMOS sensing devices. It compensates the effects of higher noise than in Charge Couple Devices (CCDs) by taking advantage of CMOS sensors capability to manage regional shutter time on small windows surrounding each viewed star. This capability allows acquired star frames to have different number of detected stars depending on the local value of the shutter time. In these conditions, sensor accuracy is equal to the temporal average of single frame accuracy. A comparative analysis of star sensitivity and sensor noise has been carried out considering both CMOS and CCD technologies. This analysis has estimated the value of the limiting detectable star brightness for different shutter times. Finally, attitude accuracy has been evaluated for each photodetector type and algorithm.

1. INTRODUCTION

The replacement of CCDs with CMOS sensing devices (Active Pixel Sensors APS) in space applications is a topic under investigation [1], [2]. CMOS technology has many advantages on CCD such as low power consumption, single voltage supply needed (3.3V or 5V), efficient handling of non stellar objects, increased radiation hardness, simplified qualification procedures, capability of local reset and readout, on-chip integrated timing, control, and analog-to-digital conversion.

Unfortunately, CMOS sensors noise performance is worse than CCDs one [3]. For this reason, the star detection threshold of CMOS sensors is set at an higher level of brightness than the one of CCDs. Since the number of observed stars decreases logarithmically with increasing brightness, current CCD devices can detect a number of stars that is considerably higher than CMOS sensors [4].

Several small satellites missions require low-cost, compact star trackers with accuracy in the range 1-3 arcsec [4]. The presented algorithm has been designed to permit useful application of CMOS devices in these applications. Indeed, attitude measurement accuracy is improved by performing a temporal average of the single frame accuracy resulting from different local shutter times on pixel windows surrounding stars with a different brightness level. Resulting accuracy performance has been evaluated by means of statistical analysis.

2. STAR SENSITIVITY ANALYSIS

A star can be detected using an image sensor array by selecting the exposure time so that the ratio of the number of photoelectrons \( n_{phe} \) to the number of background noise electrons \( n_{no} \) is higher than a stated threshold [4]. For this reason, an analysis of star sensitivity has been performed to carry out the limiting star brightness value for a specific sensor and shutter time.

2.1 Evaluation of \( n_{phe} \)

The value of \( n_{phe} \) can be estimated by means of the following expression [5]:

\[
n_{phe} = \Delta t_{sh} k_m \frac{A_{pix}}{A_{blur}} \int_{\lambda_{min}}^{\lambda_{max}} R_{op}(\lambda) \cdot \frac{Q.E.}{Q.E.} \cdot \frac{\lambda_{max}}{\lambda_{min}} \cdot A_{ap} \cdot n_{ph.} \cdot \tau_{op} \cdot M_y \cdot d \lambda
\]

where \( \Delta t_{sh} \) is the shutter time in seconds, \( k_m < 1 \) accounts for motion distortion, the ratio \( \frac{A_{pix}}{A_{blur}} \) of the pixel area to the area of the blur accounts for defocusing [4], \( \lambda_{min} \) and \( \lambda_{max} \) are the minimum and maximum wavelength in sensor bandwidth, \( \tau_{op} < 1 \) accounts for transmission loss in the optics due to
absorption and internal reflections, $R_{op}(\lambda)$ is the normalised spectral transmittance of the optics [6], $QE_{\text{max}}$ is the sensor maximum absolute quantum efficiency, $QE(\lambda)$ is the sensor normalised quantum response, $A_{ap}$ is the optics aperture area and $n_{ph,\lambda}(\lambda, T_c)$ is the number of photons per unit area, per wavelength and per second that come to the space near the Earth from a star with colour temperature $T_c$ and visual magnitude $M_v$. This spectral distribution of $n_{ph}$ has been computed assuming the blackbody radiator model for the stars [7] and considering a constant value for $T_c$ ($T_c=5700K$). Indeed, this is a conservative hypothesis because, for a given value of $M_v$, silicon sensor output is minimum at this value of temperature [8]. In addition, this is the temperature of one of the most populated star spectral classes (G2) [4]. The value of $n_{ph}$ is reported in the literature for $M_v=0$ [4],[9] the values of $n_{ph}$ for different $M_v$ have been computed by scaling the reported value.

2.2 Photodetector noise model

In figure 1 (a) and (b) are reported the block diagram of CMOS [10] and CCD [11] noise models.

![Block diagrams of CMOS (a) and CCD (b) sensors noise models](image)

**Figure 1** – Block diagrams of CMOS (a) and CCD (b) sensors noise models

It must be noted that the rms contribution of the shot noise is $<n_{\text{shot}}^2> = n_{\text{ph}} + n_{\text{dark}}$, where $n_{\text{dark}}$ is the number of dark current electrons that depends on sensor temperature and it is measured by sensor manufacturers [11]. The rms contribution of quantization noise is $<n_{\text{quant}}^2> = \frac{n_{\text{WELL}}^2}{2N\sqrt{12}}$ where $n_{\text{WELL}}$ is the number the full well electrons [11] and $N$ is the number of bits of the analog to digital converter. The $1/f$ noise of the amplifiers is usually removed by means of double sampling techniques [10]. Reset Noise $<n_{\text{reset}}^2>$, Fixed Pattern Noise (FPN) $<n_{\text{FPRNU}}^2>$ and Photo Response Non Uniformity (PRNU) $<n_{\text{PRNU}}^2>$ rms values are usually measured and documented by sensor manufacturers. The total number of noise electrons rms value $<n_{\text{no}}>$ is given by:

$$<n_{\text{no}}^2> = <n_{\text{shot}}^2> + <n_{\text{reset}}^2> + <n_{\text{FPRNU}}^2> + <n_{\text{PRNU}}^2> + <n_{\text{quant}}^2>$$

CMOS and CCD sensors have different configuration of amplifiers. Indeed, CMOS have three amplifier stages instead of one and two of them are integrated into single pixels and columns. For this reason, CMOS sensors have higher levels of dark current, FPN, and PRNU.

3. TRACKING ALGORITHM FOR SENSORS WITH REGIONAL SHUTTER

CMOS sensors can manage regional shutter time on small windows in their arrays, if they are equipped with single pixel reset circuits [2]. Since the limiting detectable star visual magnitude is a function of the exposure time, stars can be divided into classes depending on the time that is needed to detect them. The first class includes all brightest stars that can be detected in a time equal to the assigned sensor measurement update period. Subsequent classes are formed by all the stars that need a time that lies from the maximum time of the previous class to its double. The number of star detected at each measurement update depends on the classes that perform their shutter in the same instant. This condition is displayed in the timing charts in figure 2 where two and three different classes of shutter time are considered. Since attitude measurement accuracy is a function of the number of detected stars ($N_i, i=1...\text{number of classes}$), the accuracy of a sensor that adopts this algorithm is the temporal average of single frame accuracies over a period. This value is better than the single frame accuracy. Sensors that adopt this method can increase their accuracy without decreasing their update rate.
4. ATTITUDE DETERMINATION ACCURACY ANALYSIS

Two reference model of photodetectors for each kind of technology have been selected to evaluate the performance of the algorithm: CCD 77-0™ [12] model manufactured by EEV™ and STAR250™ CMOS sensor [13] manufactured by Fill Factory™. They have been selected because they have similar array and pixel size and also because they have been designed for space tracking applications.

4.1 Sensor model

Common optical characteristics have been assumed for sensor model equipped with both photodetectors. They are reported in table 1. Table 2 reports characteristics of photodetectors.

<table>
<thead>
<tr>
<th>Table 1. Sensor characteristics common to both configurations</th>
<th>Table 2. Photodetector characteristics reported by manufacturers data sheets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular FOV radius 5°</td>
<td>Fillfactory™ STAR 250™</td>
</tr>
<tr>
<td>F# ~1.1</td>
<td>EEV™ CCD 77-0™</td>
</tr>
<tr>
<td>Circular blur radius 1.5 pixels</td>
<td>Pixel array 512 x 512 pixels</td>
</tr>
<tr>
<td>Lens type 2 plano-convex</td>
<td>Pixel size 25 x 25 µm²</td>
</tr>
<tr>
<td>standard stock lenses</td>
<td>Full well capacity 311 Ke-</td>
</tr>
<tr>
<td>Dark current</td>
<td>Dark current 4750 e-/s @ room temp.</td>
</tr>
<tr>
<td>Reset noise 76 e-</td>
<td>4750 e-/s @ 293K</td>
</tr>
<tr>
<td>FPN 1σ&lt;0.1% of full well</td>
<td>Reset noise 76 e-</td>
</tr>
<tr>
<td>PRNU 1σ&lt;1.3% of signal</td>
<td>1σ&lt;0.05% of full well</td>
</tr>
</tbody>
</table>

4.2 Attitude accuracy evaluation

The solid angle \( S_{FOV} \) of the sector of sky included in a circular field of view (FOV) with angular aperture \( A \) is given by:

\[
S_{FOV} = \frac{1 - \cos \left( \frac{A}{2} \right)}{2} \tag{3}
\]

The number of stars \( N_M \) that are brighter than a the value \( M \) of the limiting star magnitude of a sensor is statistically measured by means of the expression [4]:

\[
N_M = 6.75 e^{1.08 M} \tag{4}
\]

The product \( N_{FOV} = S_{FOV} \cdot N_M \) is equal to the mean number of stars detected in a single frame. The resulting angular accuracy \( \sigma_{frame} \) of boresight direction determination is given by [4]:

\[
\sigma_{frame} = \frac{A \cdot S_{single \ star}}{N_{pixel} \sqrt{N_{FOV}}} \tag{5}
\]
where $\sigma_{\text{single star}}$ is the accuracy in the determination of the direction of a single star and $N_{\text{pixel}}$ is the number of pixels in one line of the array. For CCD standard algorithm $\sigma_{\text{frame}}$ is the sensor accuracy itself. Indeed, in the accuracy for CMOS sensor with regional shutter is the temporal average of $\sigma_{\text{frame}}$.

5. RESULTS

Limiting star magnitude have been evaluated for both photodetectors models with a +6dB detection threshold by means of the star sensitivity analysis reported in section 2. The resulting values for shutter times equal to 1s, 0.5s, 0.25s, and 0.125s are reported in Table 3.

<table>
<thead>
<tr>
<th>Shutter time</th>
<th>1s</th>
<th>0.5s</th>
<th>0.25s</th>
<th>0.125s</th>
</tr>
</thead>
<tbody>
<tr>
<td>EEV 77</td>
<td>8.9</td>
<td>8.2</td>
<td>7.5</td>
<td>6.8</td>
</tr>
<tr>
<td>STAR 250</td>
<td>7.5</td>
<td>6.8</td>
<td>6.2</td>
<td>5.4</td>
</tr>
</tbody>
</table>

The results of Table 3 can be placed into (4) and (5) to compute $\sigma_{\text{frame}}$ for the different photodetector models and exposure times. The average accuracy of APS photodetectors is computed both with no multiple shutter time and with double and quadruple shutter times at both 4 Hz and 8 Hz measurement update rate. It must be noted that the algorithm with multiple regional shutter has attained the target accuracy level of 1-3 arcseconds that is very close to CCDs one.

<table>
<thead>
<tr>
<th>Sensor model</th>
<th>No. stars</th>
<th>Accuracy (arcsec)</th>
<th>4 Hz</th>
<th>Accuracy (arcsec)</th>
<th>8 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>EEV 77</td>
<td>41</td>
<td>1.1</td>
<td>41</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>STAR 250 single shutter</td>
<td>10</td>
<td>2.2</td>
<td>19</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>STAR 250 double shutter</td>
<td>15</td>
<td>1.9</td>
<td>4</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>STAR 250 triple shutter</td>
<td>20</td>
<td>1.8</td>
<td>7</td>
<td>2.6</td>
<td></td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

An innovative tracking algorithm that increases the accuracy of star trackers based on CMOS sensors has been presented. It takes advantage of the sensor capability to manage local shutter on small windows in the array. A detailed analysis has been performed to estimate algorithm accuracy. The results have been compared to the ones of a standard algorithm applied to CCD based sensors and the same performances have turned out (1-3 arcsec accuracy at same update rate).

7. REFERENCES

7. R.C. Ramsey , Spectral Irradiance from Stars and Planets, above the Atmosphere, from 0.1 to 100.0 Microns, *Applied Optics*, 1(4), 465-471 (1962);