Computation of Rotational Elements of Planetary Bodies from Control Point Network Analysis in ICRF

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ABSTRACT

A modified approach to solve a global control point networks based on photogrammetric stereo image data has been developed. The implementation simultaneously solves for the bodies rotation parameters. Though this approach is applicable to any planetary body having sufficient data coverage, we focused on the re-analysis of the control point network for Phobos.
In particular, we analytically solve for the pole axis orientation, precession, and longitudinal librations, a capability not previously implemented in common least-squares bundle block adjustment software. Our results confirm the adopted pole axis orientation of Phobos and indicate a forced libration amplitude of -0.99 degrees.

1. INTRODUCTION

The orientation of planetary bodies is commonly described by their pole axis orientation and the relative position of the prime meridian with respect to the vernal equinox at a specific time epoch [1]. For Phobos, the recommended parameter set describes a precession with a period of about 826 days and a forced libration amplitude (small oscillation superimposed on the mean rotation rate) caused by the interaction of Phobos' body with the Martian gravity field of -0.78 degrees.

Planetary reference frames are typically realized by control point network analysis. The orientation parameters of planetary bodies are, however, not directly computed as the available software tools for the analysis operate in the body-fixed reference frame. Previously, the parameters have been determined through empirical approaches, in particular by systematically scanning through the parameter space e.g. [2]. Not surprisingly, some of the rotational parameters of Phobos, such as rotational axis orientation or precession have never been directly measured.

Earlier it was demonstrated that an analytical determination of the rotational elements based on a least-squares bundle block adjustment can be successful even with a data set covering a short time period [3]. Here a modified approach is presented, that determines the rotation parameters in inertial space directly. A new bundle block adjustment software was implemented. Another important aspect of this analysis is that the underlying mathematical model is more coherent which improves the stability and accuracy of the adjustment solution.

To verify this approach, images of the Super Resolution Camera (SRC) on board the European Mars Express (MEX) spacecraft and the Viking Orbiter (VO) camera of Phobos were used. The data set consists of measured image coordinates of conjugate points that had already been used to derive earlier versions of the control point network [4]. The SRC images were obtained between May 18, 2004 and Aug 13, 2007 thus covering the period of precession 1.4 times.

2. FORCED LIBRATION AMPLITUDE

Past results of efforts to determine the forced libration amplitude for Phobos differ significantly. Observations range from 0.79° [7] to 1.24° [4] while models assuming homogeneous mass distributions suggest amplitudes between 0.81° [8], 1.1° [4] and 1.99° [9]. Respective observations and models are usually in agreement within their error margins. However, due to the observation methods larger uncertainties cannot be ruled out.

3. LEAST-SQUARES ADJUSTMENT

A novelty of the presented approach is the formulation of the adjustment problem in the inertial framework. The function to describe the image observations is derived from equations of the form

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{Phobos} = R_2 R_1 \begin{pmatrix} x \\ y \\ f \end{pmatrix}_{Camera} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}_{Phobos}$$

$R_{1,2}$ are rotation matrices. R_1 rotates from the frame of the image coordinates (x,y,f) into the inertial frame ICRF and R_2 rotates the coordinate vector from ICRF to the IAU_Phobos body fixed frame. The lefthand vector describes a control point on Phobos and the most right vector the position of the camera within the IAU Phobos frame. Apart from the classical six unknown parameters per frame in a bundle block adjustment, in this approach the Jacobi-matrix contains additionally the derivatives of the three parameters α, δ and ω which form the rotational matrix R_2 and are defined as the time-dependent functions

$$\alpha(t) = \alpha_0 + a*\cos(M1) - 0.108\ T$$

$$\delta(t) = \delta_0 + b*\sin(M1) - 0.061\ T \qquad (1)$$

$$\omega(t) = \omega_0 + c*\sin(M1) + d*\sin(M2) + k(t)\ .$$

Table 1 provides pre- and post-adjustment values of the variables of interest, see [1] for the full definition of (1). In a first approach the derivatives $d\alpha$ (resp. $d\delta$, $d\omega$) are built with respect to the start value α_0 (resp. δ_0 , ω_0). In a second adjustment we use the updated start values in the definition of equation (1) and build the derivatives $d\alpha/da$ and $d\omega/dd$ where d is the forced libration amplitude, b and c are functions of δ_0 and a.

It should also be mentioned that all classical frame parameters were included as additional observations with weights derived from a prior bundle block adjustment in the body-fixed frame.

4. SUMMARY AND CONCLUSIONS

The original control network contained 689 points from which 680 remained after the new adjustment. The mean error of their variance-vector could be reduced to 19.14m (30m in [4] with 665 points).

Table 1: Pre- and post-adjustment values for Phobos amplitudes of time-varying orientation parameters (eq. 1)

Value	Apriori	Aposteriori
a	1.79	1.7963 ± 4.4e-05
b	-1.078	-1.079992 ± 2.1e-04
d	-0.78	-0.9915 ± 3.7e-04

The determined values for the orientation of Phobos' rotation axis do not differ significantly from those previously adopted. However, the current result shows a significant change of the forced libration, compared to its start value of –0.78 degrees. Since already small changes of the derived amplitude can effect the mass distribution models (as discussed in [5, 6]) this is consequently the base for further studies.

The software was preliminary tested with a much larger data set of Vesta observations. The tests showed that the implementation can very well cope with such large data sets and the orientation values for Vesta could be confirmed with 309.025, 42.23 within the given accuracy of 0.03 degree.

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