RESOLUTION EFFECTS IN RADARCLINOMETRY. R. L. Kirk[1] and J. Radebaugh[2], [1]U.S. Geological Survey, Astrogeology Program, Flagstaff, AZ 86001, USA (rkirk@usgs.gov), [2]Department of Geological Sciences, Brigham Young University, Provo, UT 84602, USA (jani.radebaugh@byu.edu).

Introduction: Data from the Cassini-Huygens mission, in particular images from the Cassini Titan Radar Mapper (RADAR) have revealed Saturn's giant moon, Titan to be a world whose geologic diversity and complexity approach those of the Earth itself. Estimates of topographic relief are, naturally, of enormous interest in the effort to understand the nature of Titan's surface features and quantify the processes by which they formed. Such data are available from a variety of sources, including altimetry and, increasingly, stereo imaging by the RADAR, but radarclinometry (radar shape-from-shading) has received considerable attention because it provides the highest resolution topographic measurements and can be applied to single images, wherever topographic shading dominates intrinsic variations in radar backscattering strength.

In this abstract, we attempt to explain the surprising result that the majority of topographic measurements of Titan by radarclinometry appear to be asymmetric: slopes facing the RADAR instrument tend to be areally extensive but shallow, whereas slopes facing away are limited in area but relatively steep. We describe how this is a natural consequence of the inability of the instrument to resolve the foreshortened facing slopes, causing them to be over-represented (by area, but underestimated in magnitude) when we attempt to reconstruct the surface from the image. We quantify this effect by constructing models of the imaging and reconstruction of idealized symmetrical mountains, and show that the magnitudes of slopes facing away from the instrument are estimated relatively accurately. As a result, height estimates from radarclinometry can be at least approximately corrected for the effects of limited resolution. This result is of obvious geoscientific significance for Titan: it indicates that some mountainous areas approach 2 km in local relief. Our modeling should also be useful to the interpretation of radarclinometric models of features at the limit of resolution in other SAR images, such as Magellan data for Venus, as well as current earth-based and planned orbital imaging of the Moon.

Background: The Cassini RADAR is designed to investigate Titan's surface by using 2.2 cm radiation in both passive (radiometry) and active (altimetry, scatterometry, and SAR) modes [1]. The altimetry data so far reveal departures of at most a few hundred meters from the mean radius, though the measurements cover much less than 1% of Titan and are limited in resolution by the 20–50 km beam footprint [2]. Overlapping SAR images are only now beginning to be obtained, and the analysis of these by radar stereogrammetric methods to determine topography is challenging for a variety of reasons [3]. The primary (and highest resolution) source of topographic information has therefore been radarclinometry, applied to the SAR images. These images (11 to date) typically cover ~1% of Titan, in a narrow strip that varies in width between 150–600 km and can be as long as 6000 km. The images are created with a pixel spacing of 1/256° (175.5 m), but the true resolution varies. The range resolution is typically between 400 and 600 m.

We have used both one-dimensional and two-dimensional radarclinometry to investigate the relief of features on Titan. In the former method [4,5], brightnesses along a profile in range are interpreted as slopes (based on a model of the surface backscatter cross-section σ_0 as a function of incidence angle) and integrated to yield an elevation profile. Slopes at right angles to the profile must be assumed to be zero, so this technique is limited to geometrically simple features, but it is fast and can be applied even to very small areas of uniform scattering properties. Two-dimensional clinometry [6,7] uses an iterative approach to estimate the heights at points in a rectangular grid (digital terrain model or DTM) in such a way that the observed image is reproduced. This approach is useful for mapping larger areas of uniform scattering properties and complex topography.

The first RADAR image of Titan, from the Ta flyby, revealed only a few features that might be topographic, and these were found to be only 100–300 m in height [4]. Subsequent images (T3, T8 flybys) revealed areas of unambiguous relief, similar in appearance to terrestrial mountains in SAR images, in both chains and isolated patches within Titan's plains. Our initial set of radarclinometric profiles indicated that these "mountains" were mostly rather low (<500 m) [8]. Heights from clinometry agreed well with heights calculated from the parallax distortion of the features (radar "layover", which causes facing slopes to appear compressed and back slopes to appear elongated) under the assumption that the features were intrinsically symmetric [9]. This is a form of "single-image stereo-analysis": if the feature is symmetric, the opposite-side stereo image will be the mirror reflection of the image in hand, and analysis can proceed as if both images had been obtained [10]. Estimated slopes were low, ≤10°.

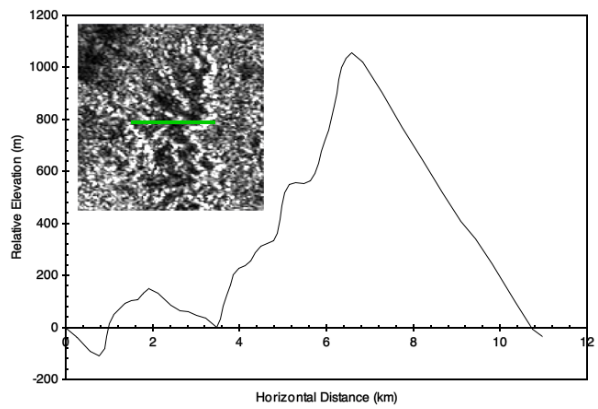

Figure 1. Example of a Titan mountain in Cassini RADAR T8 image (illumination from right) and radarclinometric profile at location in green. Note vertical exaggeration.

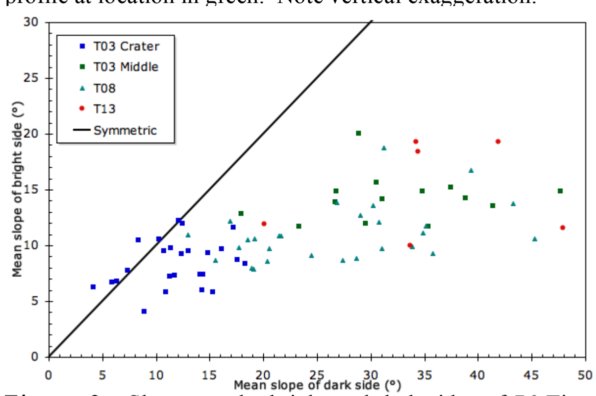

Figure 2. Slopes on the bright and dark sides of 76 Titan mountains, as estimated by radarclinometry. The average slope of a relatively straight and representative segment of several pixels is used, to avoid the bias of extreme 1-pixel slopes by noise. Note the increase of asymmetry with slope.

As additional profiles were calculated for mountain peaks and chains in T3 and T8 [11], and especially as T13 imagery revealed the continent-sized bright area Xanadu to consist largely of rugged, mountainous terrain [12], evidence mounted that (a) many mountains were significantly higher and steeper than the features described previously, (b) many were distinctly asymmetric (Figure 1), and (c) asymmetry was generally correlated with slope (Figure 2). As noted above, the sense of asymmetry is consistent, with dark back slopes exceeding radar-facing slopes in magnitude.

Explanations for Asymmetry: We have considered the following hypotheses to account for the apparent asymmetry:

Real asymmetry. The possibility that Titan's mountains are truly asymmetric must be considered seriously, particularly for Xanadu, a large region of high radar and optical reflectivity that was imaged along one edge. Perhaps Xanadu is an eroded anticlinal structure, and the RADAR is seeing outward-dipping "flatirons" rather than symmetric mountains. The many isolated mountains and chains imaged elsewhere in Titan's plains also display the same sense of asymmetry relative to the spacecraft, however, regardless of the geographic direction from which they are observed. That this is a real effect seems extremely improbable.

Speckle noise. We initially tabulated the extreme slopes (over single pixels) for each mountain. Radar speckle noise might cause some pixels to appear to be completely shadowed (no signal), but would have a smaller effect of increasing bright slopes. The asymmetry persists, however, when 90th or even 50th percentile front and back slopes are compared, or when average slopes over "representative" sections of the mountains are used. Results from two-dimensional radarclinometry, which is less noise-sensitive because it fits a surface through multiple adjacent rows of the image rather than a single profile, also show the same asymmetry. It seems unlikely to be a result of speckle.

Inappropriate backscatter model. In the raw images, mountains appear to "lean" toward the spacecraft because of layover. This distortion should be corrected in the reconstructed profiles. If the backscatter model used leads to overestimates of the height, the layover would be overcorrected, as observed. We initially used a backscatter model of the form $\sigma_0(i) = C \cot(i)$, where i is incidence angle and C is a normalizing constant chosen so the profile across an entire mountain is level. This model fits the observed behavior of Titan's plains for $2° \leq i \leq 45°$ and goes to zero for $i = 90°$ (i.e., shadows are dark). The T13 image, however, revealed that Xanadu is not only distinctly brighter than the plains (σ_0 = 1–1.5 versus 0.2–0.3), but that its cross-section varies much less strongly with incidence (in the range 12°–30°) than the "cotangent law" would suggest. A diffuse scattering law, e.g., of the form $\sigma_0(i) = C \cos(i)$ is therefore more appropriate. The SAR image and available scatterometry data do not sample Xanadu at small incidence angles, so it is not known if a quasi-specular scattering component is present to enhance the return at these small angles. The lack of polarization of thermal emission from Xanadu suggests that such a quasi-specular surface reflection is likely to be very weak [13]. Isolated mountains cannot be studied over as broad a range of incidence angles as Xanadu, but as an ensemble they span the range 15°–30°. Backscatter cross-sections for these mountains are very similar to those for Xanadu. The results of substituting a diffuse "cosine law" for the more strongly varying "cotangent law" in radarclinometry calculations are roughly to double the estimated heights of all mountains and to *increase* their asymmetry. The problem is thus worse than it initially seemed.

Resolution. We claim that the most likely explanation for the apparent asymmetry is that the SAR images do not fully resolve the foreshortened, bright sides of Titan's mountains. It is intuitively clear that this effect will lead to heights calculated geometrically, on the assumption of symmetry, to be underestimated. Higher, steeper mountains are more laid over in the images; if blurring increases the apparent width of the bright side (not because it is bright, because it is narrow) then the estimated height will be reduced. Blurring of the bright slope also seems likely to lead to its width being overestimated when radarclinometry is applied. To quantify this effect, we turn to simple models of the imaging and reconstruction processes.

Idealized Mountain Models: We consider the simplest possible geometry for a mountain, with straight sides of equal absolute slope facing toward and away from the radar. The feature is assumed to extend indefinitely (or, at least, much farther than the azimuth resolution) along the radar track, so slopes in this direction can be neglected. The cross-sectional shape of the mountain in a plane of constant azimuth is thus an isosceles triangle (Figure 3). The total energy returned to the radar from each side of the mountain can be calculated from the area (length) of the side and σ_0 at the appropriate local incidence angle. The strength of the received signal (energy per pixel, i.e., per range bin) is then proportional to this total energy divided by the distance in range that the side occupies. If the range spanned by the bright side is smaller than the assumed resolution of the radar system, then the signal from the apparent bright side consists of the actual bright slope return plus the energy from the portion of the dark slope that fills out the minimum resolvable range. If the slope of the mountain exceeds the incidence angle (measured at a horizontal surface), the bright face will be not merely laid over, but folded over. The apparent signal from the bright face then consists of the energy from the face itself plus contributions from the area in front of and behind this face that lie at the same ranges from the radar. Careful bookkeeping therefore allows us to calculate both the brightness and apparent width of the brighter and darker parts of the mountain in all cases: resolved, unresolved, unresolved and folded over, or folded over so severely that the folded area exceeds the resolution limit. The calculated brightnesses and widths are then used to reconstruct the shape of the mountain. In this process (as in our radarclinometry profiling with real data) no assumption of symmetry is made, but it is assumed that the bright slope is not steep enough to be folded over. The available information in our ideal model does not provide any means of distinguishing between the folded and non-folded cases, so this assumption is necessary to obtain a unique result.

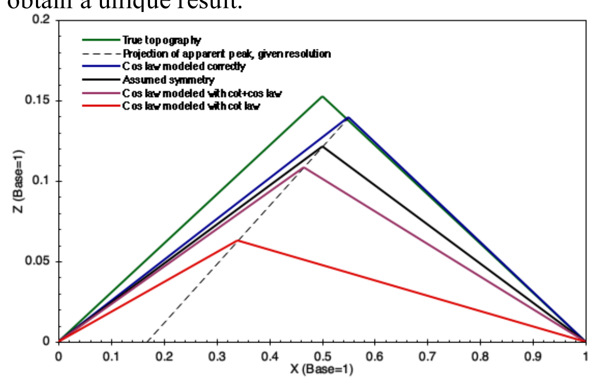

Figure 3. Idealized model of a symmetric mountain, seen in cross-section in a plane of constant radar azimuth. Radar illumination is from the left. The figure shows the true profile (green) and profiles recovered under the assumption of symmetry and by radarclinometry with various scattering laws. In this case (i = 20°, θ = 16°, resolution = 1/6 of mountain width) the bright face is unresolved but is not folded over. These profiles illustrate one case drawn from the parametric plots in Figs. 4–5.

The parameters of the model are the incidence angle on the horizontal surface, the slope of the mountain, and the resolution in ground range, which can be expressed as a fraction of the basal width of the mountain. The incidence angle affects the particular slopes at which the bright face becomes unresolved and folded over, but the behavior is qualitatively similar so we present results for a representative incidence angle of 20°. A ground range resolution of 1/6 the width of the mountain may also be taken as typical; resolutions range from 450–600 m and mountain widths of 3–5 km are commonplace [11]. Finally, we consider the effects of various scattering laws, including cases where the law assumed in the radarclinometric reconstruction is not the same as the "actual" scattering behavior assumed in creating the image.

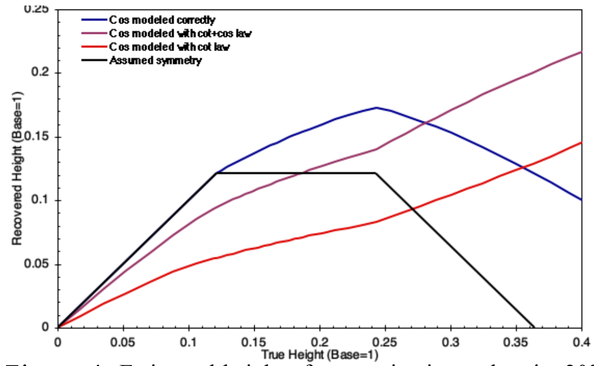

Figure 4. Estimated height of mountains imaged at $i = 20°$ as a function of true height, as obtained by assumed symmetry and by radarclinometry with correct and incorrect scattering law assumptions.

Results: The most general and important result from our models is the confirmation that failure to resolve the bright side leads to asymmetry in the observed sense (leaning away from the radar) in all cases where the correct scattering law is used in the reconstruction step. In almost all such cases, the height of the feature is underestimated (Figure 4), both when the bright face is folded over and when it is merely unresolved. The exception (not shown) is that, for scattering laws that peak sharply at low incidence angles, the height can be overestimated when the bright face is illuminated almost perpendicularly ($\theta \sim i$) and is therefore very bright.

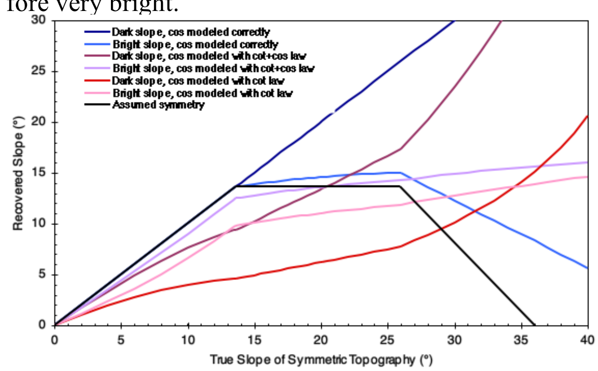

Figure 5. Estimated bright and dark side slopes of mountains imaged at $i = 20°$ as a function of true slope, for same modeling assumptions as Fig. 4. Note that the dark side slope is recovered accurately when the correct scattering law is used (dark blue line).

The slope on the bright side is generally underestimated. For the diffuse cosine law, the dark slope is recovered almost exactly in all situations (Figure 5), a very useful result for interpreting the results with real data. For more strongly varying backscatter models (e.g., the cotangent law or any of various well known quasi-specular scattering laws, alone or with a diffuse component added) the dark slope can be overestimated when the bright face is illuminated almost perpendicularly, but is otherwise close to its true value.

Finally, the effect of an incorrectly assumed scattering law depends on how strongly the true and assumed laws vary with incidence angle. If the true law is weakly varying, like the cosine law used in Figs. 3–5, image contrast will be low and radarclinometry with a more strongly varying law will underestimate heights and slopes (even in the resolved case). As discussed previously, underestimation of heights leads to under-correction of parallax distortion, yielding profiles that "lean" toward the radar (cf. violet and red profiles, Fig. 3). As the slope increases, this asymmetry reverses because of resolution effects, giving the "usual" case of mountains leaning away from the radar. These results are entirely consistent with the increase in heights, slopes, and asymmetry that we observed when we recalculated our profiles with the cosine rather than cotangent scattering law. Conversely, if the assumed law varies too weakly, heights and slopes will be overestimated, and the asymmetric lean away from the radar will be exaggerated, even in the resolved case.

Conclusions: The modeling results described above support our interpretation of the apparent asymmetry of Titan's mountains as arising from the limited resolution of the Cassini RADAR in relation to the foreshortened size of these relatively small features. The available evidence suggests a relatively diffuse scattering law for Xanadu and for isolated mountains and mountain chains but cannot rule out the presence of a quasi-specular enhancement at small incidence angles. Such an enhancement seems unlikely based on the observed distribution of bright and dark-side slopes, however. It would lead (in our models, which are calculated with a strictly diffuse law) to a noticeable asymmetry even for mountains with low and fully resolved slopes. This is not seen in Fig. 2.

Because the dark-side slope can be estimated very accurately for the cosine law (and usually quite accurately for other scattering laws), the true height of a triangular mountain can be calculated simply from this slope, the observed basal width, and the knowledge of symmetry. Real mountains are more complex in form, with slope variations on each side and even subsidiary peaks. Nevertheless, applying the idealized correction from radarclinometric height to true height to real profiles can be expected to improve the accuracy of height estimates. Figure 6 shows the results of such a correction for Titan. Mountains more than 1 km in height are seen to be relatively common, with some reaching nearly 2 km. Note that, although results of only 5 profiles in Xanadu are shown, these are representative of extensive areas of rugged peaks. We look forward to the availability of stereo images of mountains with which to verify our radarclinometric estimates.

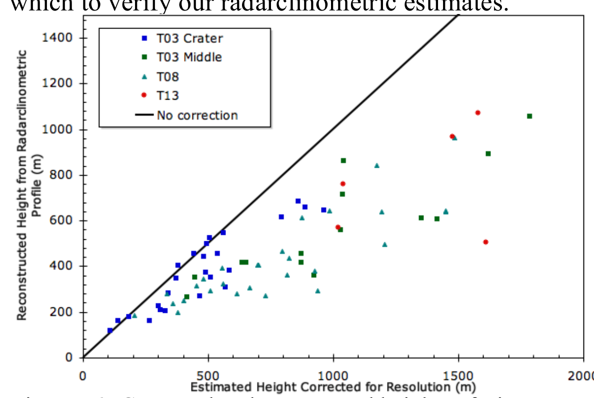

Figure 6. Corrected and uncorrected heights of Titan mountains. The correction factor is based on the assumption that dark slopes have been estimated correctly, as is the case for our simple mountain models.

In closing, we note that this study provides a potentially surprising example of what can be learned even when the features of interest are only partially resolved by the available remote sensing data—a situation that arises all too frequently in planetary investigations.

References: [1] Elachi, C., et al. (2004) *Space Sci. Rev., 115*, 71. [2] Callahan, P., et al. (2007) *JGR*, submitted. [3] Kirk, R.L., et al. (2007) this workshop. [4] Kirk, R.L., et al. (2005) *LPS XXXVI*, 2227. [5] See Soderblom, L.A., et al. (2002) *LPS XXXIII*, 1254 for the equivalent technique applied to optical images. [6] Kirk, R.L. (1987) Ph.D. Thesis, Caltech, 165–258. [7] See Kirk, R.L., et al. (2003) online at http://astrogeology.usgs.gov/Projects/ISPRS/Meetings/Houston2003/abstracts/Kirk_isprs_mar03.pdf for more on the optical equivalent. [8] Radebaugh, J., et al. (2006) *LPS XXXVII*, 1007. [9] Weitz, C.M. (1993) in *Guide to Magellan Image Interpretation*, JPL Pub. 93-24, 85. [10] Plaut, J. (1993) in JPL 93-24, 33. [11] Radebaugh, J., et al. (2007) *Icarus*, in revision. [12] Kirk, R.L., et al. (2006) *AAS Bull., 38*, 52.03. [13] Janssen, M.A., et al. (2006) *AAS Bull., 38*, 56.06.