Fault detection filter design

In the model based fault diagnosis, the fault detection task is achieved by detecting discrepancies between the outputs of the monitored aircraft and the predictions obtained with a mathematical model. These discrepancies - called also *residuals* - are indications of faults and are produced by special devices called *residual generators*. The residual generators (or fault detection filters) are physically realizable systems having as inputs the measured outputs and the control inputs of the monitored system, and as output the generated residual signal.

A fault detection (FD) filter has at least two basic functions: (1) generating zero residuals in the fault-free case; (2) generating nonzero residuals when any fault occurs in the system. A more advanced functionality, like fault detection and isolation (FDI) by an exact location of faults, can be often achieved by designing a bank of fault detectors or by direct design of FDI filters. For an efficient implementation and operation, it is generally desirable to keep the order of the fault detector(s) as low as possible. Thus, the design of fault detectors with possible least dynamical orders is an important practical aspect.

Consider the linear time-invariant system described by the input-output relations

$$y(s) = G_u(s)u(s) + G_f(s)f(s) + G_d(s)d(s)$$

where $y(s)$, $u(s)$, $f(s)$, $d(s)$ are Laplace-transformed vectors of the system output vector $y(t)$, plant input vector $u(t)$, fault signal vector $f(t)$ and disturbance vector $d(t)$, respectively, and where $G_u(s)$, $G_f(s)$ and $G_d(s)$ are the transfer-function matrices from the corresponding plant inputs to outputs.

A FD filter of least dynamical order is sought having the general form

$$r(s) = R(s)\begin{bmatrix} y(s) \\ u(s) \end{bmatrix}$$

such that $r(t) = 0$ when $f(t) = 0$ and $r(t) \# 0$ when $f_i(t) \# 0$. Besides the requirement that the TFM of the detector $R(s)$ has least possible McMillan degree, it is also necessary, for physical realizability, that $R(s)$ is a proper and stable TFM.

An FDI filter must satisfy the more stringent condition $r_i(t) \# 0$ when $f_i(t) \# 0$ and ideally the i-th component of the fault should influence only the i-th component of the residual.

Transcribing mathematically the conditions for FD, we get

$$R(s)\begin{bmatrix} G_u(s) & G_d(s) \\ I & 0 \end{bmatrix} = 0$$

and

$$R(s)\begin{bmatrix} G_f^{(i)}(s) \\ 0 \end{bmatrix} \neq 0$$

It appears that *R(s)* is a left annihilator of a certain rational matrix *G(s)*, thus one possibility to determine *R(s)* is to compute first a left minimal nullspace basis $N_L(s)$ of *G(s)*, and then to build a proper rational and stable detector as $R(s) = X(s)\, N_L(s)$, representing a linear combination of the rows of $N_L(s)$ .To obtain a least order detector, special techniques have been proposed in [3,4] to determine *X(s)* using minimal dynamic covers techniques. The above approach for the design of FD filters is based on a recent algorithm to compute minimal rational nullspace bases [1]. This computation relies on an orthogonal reduction of the system pencil matrix to a special (so-called Kronecker-like) form from which a state-space realisation of the null space basis can be constructed just by inspection. The main advantage of using rational bases (instead of traditionally used polynomial bases) is the increased numerical reliability of the computations, which allows the applicability of this method to relatively high order systems.

For the design of FDI filters, a new method based on solving linear rational matrix equations has been proposed in [2]. In this approach, the FDI filter *R(s)* is computed as the least order proper and stable solution of the linear equation

$$R(s)\begin{bmatrix} G_u(s) & G_d(s) & G_f(s) \\ I & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & M(s) \end{bmatrix}$$

where *M(s)* is a diagonal TFM chosen such that *R(s)* has the desired properties. To compute *R(s)*, the system pencil corresponding to the coefficient matrix is reduced to a Kronecker-like form which provides a complete parameterization of all stable and proper detectors. This parameterisation allows to determine least order FDI filters by solving additionally minimal order dynamic cover problems. Prototype software in MATLAB has been implemented to design FD and FDI filters. The MATLAB implementations of the corresponding filter design functions fd and fdi are based on recently implemented computational functions of the Descriptor System Toolbox: **snull** – to compute right nullspace of a rational matrix;

slrsol – to solve rational matrix equations; and **smcover2** – to compute minimal order dynamic cover.

Related publications:

[1] Varga, A.:
On computing least order fault detectors using rational nullspace bases. Proc. of IFAC Symp. SAFEPROCESS'03, Washington DC, USA, 2003.

[2] Varga, A.:
New computational approach for the design of fault detection and isolation filters. In M. Voicu (Ed.), "Advances in Automatic Control", vol. 754 of The Kluwer International Series in Engineering and Computer Science, Kluwer Academic Publishers, pp. 367-381, 2003.

[3] Varga, A.:
Reliable algorithms for computing minimal dynamic covers. Proc. of CDC, Maui, Hawaii, 2003.

[4] Varga, A.:
Reliable algorithms for computing minimal dynamic covers for descriptor systems. Proc. of MTNS, Leuven, Belgium, 2004.

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