

## Rational and Polynomial Matrices

Each rational matrix  $R(\lambda)$  can be seen as the transfer-function matrix of a continuous- or discrete-time descriptor system. Thus, each  $R(\lambda)$  can be equivalently realized by a descriptor system quadruple  $(A - \lambda E, B, C, D)$  satisfying

$$R(\lambda) = C (\lambda E - A)^{-1} B + D,$$

where  $\lambda = s$  or  $\lambda = z$  for a continuous- or discrete-time realization, respectively. It is widely accepted that most numerical operations on rational or polynomial matrices are best done by manipulating the matrices of the corresponding descriptor system representations. Many operations on standard matrices (such as finding the rank, determinant, inverse or generalized inverses, nullspace) or the solution of linear matrix equations have natural generalizations for rational matrices. The conjugate transposition of a complex matrix generalizes to the conjugation of a rational matrix, while the full-rank, inner-outer, and spectral factorisations can be seen as generalizations of the familiar LU, QR, and Cholesky factorizations, respectively. Many problems for scalar polynomials and rational functions (poles and zeros, minimum degree or normalized coprime factorizations, and spectral factorization) have nontrivial extensions to polynomial and rational matrices.

**System inversion** techniques play a fundamental role in several areas such as control theory, filtering, fault detection and coding theory. The computation of inverses and generalized inverses of rational matrices can be equivalently formulated as the computation of descriptor system realizations whose transfer-function matrices are these inverses. Explicit inverses can be computed only for full row/column rank rational matrices. Therefore, to compute various types of generalized inverses special algorithms are necessary. To avoid the numerically questionable direct manipulations of polynomial and rational matrices, numerically reliable descriptor systems algorithms based on orthogonal reduction of the system pencil to Kronecker-like forms [1] have been developed. Numerically reliable algorithms to compute various inverses of rational matrices (e.g., weak- or (1,2)-inverses, pseudo-inverses, etc.) have been proposed in [1,2]. The algorithms to compute (1,2)-inverses are able to place the resulting spurious poles arbitrarily. Thus stable generalized weak-inverses can be computed whenever they exist. The weak-inverse plays an important role in computing the inner-outer factorizations of full row rank rational matrices [3]. The computation of an appropriate left-inverse is the first step in designing residual generators for dynamic inversion based fault detectors.

Many factorization problems of rational matrices can be best solved numerically by employing descriptor systems algorithms. The **inner-outer factorization** and **J-lossless-outer factorization** play important roles in solving many control related computational problems like the stochastic balancing truncation based model approximation, polynomial or rational spectral factorizations, or H-infinity synthesis problems. The inner-outer factorization problem for arbitrary rational matrices can be solved in the most general setting by using the algorithms proposed in [4] for continuous-time systems or in [5] for discrete-time systems. For the full row rank case, the more efficient algorithms proposed in [3] can be employed. This algorithm has been extended to solve the J-lossless-outer factorization problem in [6].

**Coprime factorizations** of rational matrices play an important role in solving several categories of model reduction problems (e.g., for unstable systems, frequency-weighted, for descriptor systems). Recursive algorithms based on a Schur approach to pole assignment have been proposed to compute various coprime factorizations: with *stable factors* [8], with *inner denominator* [2,8] or *J-lossless denominator* [6,8,9], or with *proper factors* [8]. The so-called **normalized coprime factorization** of an arbitrary rational

matrix can be computed using the algorithms proposed in [4,5] or [10]. Note that this problem has been solved in the most general setting in [10].

**Nullspace** computation of rational matrices is a basic tool to solve the fault detection problem for linear time-invariant systems. The recently developed algorithm to compute a minimal rational nullspace basis [11] relies on orthogonal reduction of the system pencil to a Kronecker-like form. In conjunction with minimal cover design techniques [12], this algorithm allows to solve the least order fault detection (FD) problem in the most general setting. The solution of **rational systems of equations** is the main ingredient to address the design of fault detection and isolation (FDI) filters. The algorithm proposed in [13] allows to compute the least McMillan degree solution of a linear rational equation by combining orthogonal pencil reduction algorithms and minimal cover design techniques.

For most of above mentioned algorithms for manipulation and factorization of rational matrices robust numerical implementations are available in the [DESCRIPTOR SYSTEMS Toolbox](#) for MATLAB [14].

#### **Related Publications:**

[1] Varga, A.:

**Computation of Kronecker-like forms of a system pencil: applications, algorithms and software.** Proc. of IEEE International Symposium on Computer Aided Control System Design, CACSD'96, Dearborn, MI, CACSD '96, pp. 77-82, 1996.

[2] Varga, A.:

**Computation of inner-outer factorizations of rational matrices.** IEEE Transaction on Automatic Control, vol. 43, pp. 684-688, 1998.

[3] Varga, A.:

**On computing generalized inverse systems using matrix pencil methods.** Proc. of Mathematical Theory of Networks and Systems MTNS'2000, 2000.

[4] Oara, C., Varga, A.:

**Computation of general inner-outer and spectral factorizations.** IEEE Transactions on Automatic Control, vol. 45, pp. 2307-2325, 2000. (preprint)

[5] Oara, C., Varga, A.:

**The general inner-outer factorization problem for discrete-time systems.** Proc. of European Control Conference, ECC'99, Karlsruhe, paper F1018-8, 1999.

[6] Varga, A., Katayama, T.:

**Computation of J-inner-outer factorizations of rational matrices.** Int. J. of Robust and Nonlinear Control, vol. 8, pp. 245-263, 1998.

[7] Varga, A.:

**A Schur method for computing coprime factorizations with inner denominators and applications in model reduction.**

Proc. of American Control Conference, ACC'93, San Francisco, USA, pp. 2130-2131, 1993.

[8] Varga, A.:

**Computation of coprime factorizations of rational matrices.** Linear Algebra and Its Applications, vol. 271, pp. 83-115, 1998.

[9] Oara, C., Varga, A.:

**Minimal degree coprime factorization of rational matrices.** SIAM Journal on Matrix Analysis and Applications, vol. 21, pp. 245-278, 1999.

[10] Varga, A.:

**Computation of normalized coprime factorizations of rational matrices.** Systems & Control Letters, vol. 33, pp. 37-45, 1998.

[11] Varga, A.:

**On computing least order fault detectors using rational nullspace bases.** Proc. of IFAC Symp. SAFEPROCESS'03, Washington DC, USA, 2003.

[12] Varga, A.:

**Reliable algorithms for computing minimal dynamic covers for descriptor systems.** Proc. of MTNS, Leuven, Belgium, 2004.

[13] Varga, A.:

**Computation of least order solutions of linear rational equations.** Proc. of MTNS, Leuven, Belgium, 2004.

[14] Varga, A.:

**A DESCRIPTOR SYSTEMS Toolbox for MATLAB.** Proc. of IEEE International Symposium on Computer Aided Control System Design, CACSD'2000, Anchorage, Alaska, 2000.

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